Example 1: **Gradients, unit vectors, angles starting with a scalar field**

Given: $T = 3x^2 y - 2y \qquad k = 1 \qquad \vec{Q} = -k\vec{\nabla}T$ (obviously I'm omitting units!)

$$\vec{\nabla}T = (6xy)\hat{\imath} + (3x^2 - 2)\hat{\jmath}$$

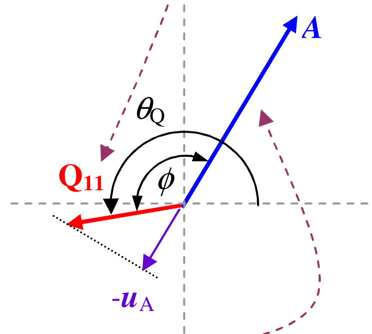
$$\vec{Q} = (-6xy)\hat{\imath} + (2 - 3x^2)\hat{\jmath}$$

$$\vec{Q}_{1,1} = -6\hat{\imath} - 1\hat{\jmath} \qquad \left|\vec{Q}_{1,1}\right| = \sqrt{37} \approx 6.0828$$

ALWAYS use the components to draw it!

$$\theta_Q = \mathrm{a}\tan\left(\frac{Q_y}{Q_x}\right) \quad \text{so} \quad \theta_{Q_{1,1}} = \mathrm{a}\tan\left(\frac{-1}{-6}\right) = 189.46° \text{ (check on calculators!)}$$

(your calculator incorrectly said it was +9.46°… but why?)



→ How much of $\vec{Q}_{1,1}$ is in the $\vec{A}$ direction, given $\vec{A} = 6\hat{\imath} + 10\hat{\jmath}$?

$$\vec{u}_A = \frac{\vec{A}}{A} = \frac{6\hat{\imath} + 10\hat{\jmath}}{\sqrt{136}} \approx 0.5145\hat{\imath} + 0.8575\hat{\jmath}$$

$$\text{as a check: } \left|\vec{u}_A\right| = \sqrt{0.5145^2 + 0.8575^2} = 1.000$$

$$\text{also, } \theta_A = \mathrm{a}\tan\left(\frac{A_y}{A_x}\right) = \mathrm{a}\tan\left(\frac{0.8575}{0.5145}\right) = \theta_A = 59.036°$$

$$\boxed{\text{We want } \vec{u}_A \cdot \vec{Q}_A} = u_{Ax}Q_x + u_{Ay}Q_y + u_{Az}Q_z = (0.5145)(-6) + (0.8575)(-1) = \boxed{-3.9445}$$

Or, done another way:

$$\vec{u}_A \cdot \vec{Q}_A = \left|\vec{u}_A\right|\left|\vec{Q}_A\right|\cos\phi = (1)(6.0828)\cos(189.46° - 59.036°) = -3.9445$$

Example 2: Given $\phi = 3 - 4x + 12y + x^2 + Ay^2 - xy$.

Find "A" such that $\nabla^2\phi = 0$. Then, find $\vec{v}$ and $\vec{\nabla}\cdot\vec{v}$.

$$\frac{\partial\phi}{\partial x} = -4 + 2x - y \qquad \frac{\partial^2\phi}{\partial x^2} = 2$$

$$\frac{\partial\phi}{\partial y} = 12 + 2Ay - x \qquad \frac{\partial^2\phi}{\partial y^2} = 2A$$

$$\nabla^2\phi = \frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial x^2} = 2 + 2A = 0 \qquad \text{so} \qquad \boxed{A = -1.}$$

Also, since $\vec{v} = \vec{\nabla}\phi$, then $\boxed{v_x = (-4 + 2x - y),}$ and $v_y = (12 + 2Ay - x) = \boxed{v_y = (12 - 2y - x)}$

$$\text{then} \quad \vec{\nabla}\cdot\vec{v} = \frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = (2) + (-2) = \boxed{\vec{\nabla}\cdot\vec{v} = 0}$$