

## Summary of Using Free Body Diagrams.

- 1. Choose an object or “body”.** You must do it in writing. The most common error is changing from one body to a similar but different, nearby body in the middle of step 4.
- 2. Draw an outline of *just* this body.** Do not draw any other objects for context. So, don't draw the ground, for example. Avoiding the context is what makes this a “free” body diagram.
- 3. Draw a coordinate system** so that you can get signs right. Directions matter, but not your choice of origin. You need six directions, but four of them can be inferred from the first two ( $\hat{x}$  and  $\hat{y}$ ): for example,  $\hat{z} = \hat{x} \times \hat{y}$ . The remaining directions are rotational directions (needed for torque), where the right-hand rule tells you the positive direction of rotation for each axis direction.
- 4. Draw and name the forces that act *on* this object** (and never draw forces exerted *by* this object). Forces are usually variants of weights, normals, tensions, and frictions. The only weight permitted is the weight of the body named in step 1. Forces must be drawn at the place where they actually act. Forces either push, pull, or scrape. A pushing force is drawn aimed *into* your objects outline. A pulling force is aimed away from it, and a scraping force is drawn “along” it.

Weight *pulls* on the center of mass (as opposed to pushing). Since normal force is *always* created by touching, they are drawn *pushing* perpendicular to the surface of the object that is touched. Friction is usually a scraping force. Sometimes weights and pressures are *distributed* over some surface. If you know how they are distributed, draw them that way instead of using a single arrow!

**5. Replace “diagonals” with components:** Review each force you've drawn. If it is not already drawn in a coordinate direction (see your choices in step 3), then start over at step 2. Do not try to “save time” by crapping up your already perfect diagram with scribbling and pathetic repairs. Start over at step 2!!! This will cost you less than 10 total seconds yet save you hours. This time, for each force that was not already in a coordinate direction, replace it with its components. For example,  $T$  might become the components  $T\sin\theta$  and  $T\cos\theta$ .

**6. Write N2L in the (as many as 6) coordinate directions that matter,** starting with  $\Sigma F_x = ma_x$ . Just below each *force* equation, copy the names of forces from your diagram. So, below “ $\Sigma F_x$ ”, write all the forces that act in the +x direction. If a force acts in the -x direction, write it with a preceding minus sign. Every drawn force will appear exactly once in these three equations.

For each torque equation, first choose a location where you are computing torque (for example, add a point “A” to your diagram). Generally engineers use  $M$  for torque, so the 6<sup>th</sup> equation will be  $\Sigma M_{Az} = I\alpha_x$ . Just below “ $\Sigma M_{Ax}$ ”, write the torques that would generate rotation, about that axis, at that point. A torque is always a product of a named force from your diagram, a perpendicular distance, and a sign. Each drawn force may appear in (up to) two torque equations in addition to its own force equation. No  $z$ -force may appear in a  $z$ -torque equation, and so on.

**7. Algebra/Solve.** Presumably you know how to do this part already.

A hard example for fluids is shown on the reverse side. The reason it's hard is thinking of a good step 1. Other choices can work, but will require a LOT more work!

## Summary of Using Free Body Diagrams.

A stone dam rests on a muddy river bed, holding back a river. When the dam is “full”, determine the required net frictional force from the ground on the dam, the net normal force from the ground on the dam, and the average horizontal position of the normal force from the ground. The specific gravity of the stone is  $SG = 4.0$ .

The dam is  $w = 5.0$  m into the page.

**Step 1:** My object is everything inside the red outline.

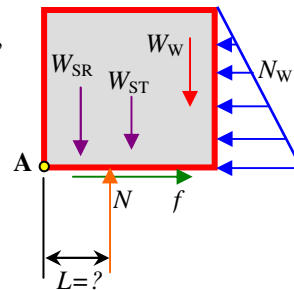
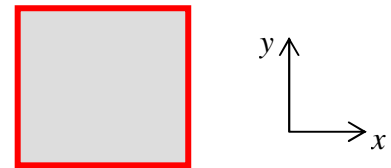
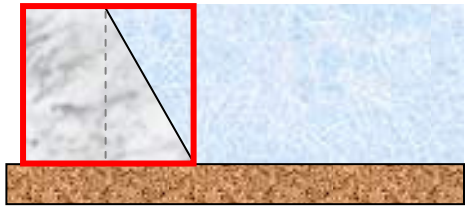
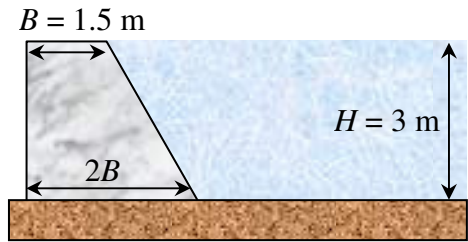
**Step 2:** Here’s just the outline. Notice that I don’t draw the remaining water, or the ground.

**Step 3:** You can see my  $x$ - $y$  coordinate system to the right...

**Step 4:** My object has some stone weight, some water weight, some normal forces from the remaining water, a normal force from the ground, and a friction force from the ground. I separated the weight of the stone into the part from the triangle, and the part from the rectangle.

**Step 5:** I have no diagonals this time!

**Step 6:** We infer from the text that the dam is at rest, so each  $a$  and each  $\alpha$  is zero... This time, we only need  $\Sigma F_x$ ,  $\Sigma F_y$ , and  $\Sigma M_z$ . (that’s generally true for any FBD that can be drawn in a plane).



$$\begin{array}{l} \Sigma F_x = 0 \\ -N_W + f = 0 \\ f = +N_W \end{array} \left| \begin{array}{l} \Sigma F_y = 0 \\ +N - W_{SR} - W_{ST} - W_W = 0 \\ N = W_{SR} + W_{ST} + W_W \end{array} \right. \left| \begin{array}{l} \Sigma M_{Az} = 0 \\ 0 \cdot f + L \cdot N - (\frac{1}{2}B)W_{SR} - (B + \frac{1}{3}B)W_{ST} - (B + \frac{2}{3}B)W_W + \frac{1}{3}HN_W = 0 \end{array} \right.$$

Various miscellaneous facts:  $N_W = \frac{1}{2}P_{\max}A_{\text{vert}} = \frac{1}{2}\rho g H(wH) = 220.5$  kN.

$$W_{SR} = (SG\rho)(BHw)g = 882.0$$
 kN

$$W_{ST} = (SG\rho)(\frac{1}{2}BHw)g = 441.0$$
 kN

$$W_W = (\rho)(\frac{1}{2}BHw)g = 110.25$$
 kN

Algebraic Results:

$$f = 220.5$$
 kN

$$N = 1433.25$$
 kN

$$L = 1.115$$
 m

Interpretation: Note that  $L$  is somewhat less than  $B$ , which makes sense, because so much of the weight of the stone is located there.