Part J: Musical Instruments

Chapter 188. Controlling Pitch

The desired frequencies for making music are explored in Chapter 87. Here we consider how to design musical instruments in order to obtain those desired pitches. There are generally two purposes for varying the frequency of a musical instrument. The first is to **tune** multiple instruments (or multiple parts of one instrument) to each other, so that they are all capable of playing from the same scale. That requires small frequency adjustments, but they need not be made rapidly. The second purpose is to **play** music, creating a comparatively rapid sequence of pitches of discrete frequencies. A third, less prominent purpose is to make music with pitches that differ by less than the smallest scale interval (as with bending notes or microtonal music), but that will not be covered in this book.

Of all the musical instruments that you are likely to run into, very few were actually invented based on scientific principles. The large majority of instruments have evolved over centuries or even millennia. There are a few mainstream instruments for which a specific inventor can be identified (e.g., the saxophone and the glass armonica). But even in these cases, their invention surely involved as much artistry as science.

The goal of this book is to identify general principles of operation. There are many interesting musical instruments with fascinating special characteristics. But in the simplifying spirit of physics, this book will instead focus on the common themes that are shared by many instruments, rather than focusing on their special features.

Non-Standing-Wave Instruments

Given any physical phenomenon that has a preferred frequency, there is likely a musical instrument based on it. This book concentrates on instruments based on standing waves. But in this few paragraphs, we acknowledge that there are instruments that use other methods. The lowly mass on a spring finds realization in the thumb piano. Its **reeds** are narrow rectangles of thin metal, which spring back to their equilibrium position after being plucked. Tuning forks can be understood in a similar way, although the cooperative motion of the two tines is a bit more complex. Harmonicas and accordions also derive their frequencies from vibrating reeds with a mass and restoring force, although they are caused to vibrate by air instead of direct contact. In all of these examples, it is not really possible to separate the mass from the spring, and closer inspection reveals other complexities. But the same underlying oscillation concepts are there. Increasing the thickness of a reed increases both the mass and the spring constant. Increasing the length of a reed increases the mass, but decreases the spring constant.

Another example of a mass on a spring is the Helmholtz resonator described in Chapter 28. Its preferred frequency depends on the volume of a body of air and the shape of a neck of air. An ocarina is an instrument based on this underlying principle. It is actually more complicated, because of the multiple holes into the body. But frequency is controlled by effectively changing the area of the "neck."

Standing Wave Instruments

There are a large number of musical instruments that rely on standing waves to define their frequencies. In most cases those waves are one-dimensional. The medium might be a string or chord, for **string instruments,** or air in a tube, for **wind instruments.** There are many commonalities between those two categories. Taking the physics approach of applying a simple model to the widest possible range of situations, this book will call them collectively **1D instruments.** Chapters 179 and 181 show that a uniform 1D medium vibrates at specific frequencies, as given by Eqs. 179.1 and 179.2 or Eqs. 181.1–181.3. Those equations, which apply equally well for strings and tubes, have the general form

$$f = (n \text{ or } m)\frac{s}{2L} \quad . \tag{188.1}$$

This tells us that there are three basic ways to control the frequency, based on the three variables in the equation. The equation only directly applies when the 1D medium is uniform along its length and when its ends are either fixed or free. But even for 1D instruments that depart from those restrictions, the underlying concepts of wave speed, length of medium, and mode (i.e., number of loops) continue to control the options.

Chapters 189–191 consider each of the three variables, for both the feasibility of adjustment, and whether it offers the necessary flexibility in pitch. There may be different answers for the purposes of **tuning** (small pitch adjustments) and **playing** music (getting the notes of a scale).

For all wind instruments, production of sound starts at one end of the 1D medium. This book will call that end the **mouthpiece,** even though in certain cases (e.g., bagpipes) the mouth is not directly applied there. The opposite end is called the **bell** when the tube flares out (and sometimes when it doesn't), and is otherwise called the **foot.** A bell does have a function that this book does not discuss this in further detail: it improves the connection between the interior of the instrument and the external air, allowing more sound power to radiate from the instrument.

Chapter 189. Changing Wave Speed

The first parameter in Eq. 188.1 that we'll consider is the wave speed *s*. For wind instruments, this is of no use at all. The speed of sound in a gas may vary with properties of the gas such as temperature (see Chapter 120), but these can't be adjusted in any practical way, either to produce music or to tune an instrument. We are stuck with the speed of sound in air.

For string instruments, the wave speed can be controlled. Considering Eq. 119.1, there are two parameters involved, linear mass density and tension. It is not possible to vary the string's linear mass density rapidly. Mass densities can be chosen to be convenient, so that the desired pitches can be obtained with reasonable tensions, lengths, etc. This is why instruments with multiple strings (like a guitar and piano) usually have strings of different thicknesses. But this parameter is not useful for either tuning or playing.

It is *possible* to vary the string tension on the fly, but it is difficult to do so with precision. The washtub base is the only instrument this author can think of that uses varying tension to obtain notes on a scale. For the most part, tension is useful for tuning. In fact, since it is usually not convenient to make long-term changes the other parameters, tension is the one best suited to tuning strings.

Chapter 190. Changing Mode

When an instrument is played, multiple modes of vibration are always active simultaneously. If that were not the case, if instruments produced only pure tones, then all instruments would sound exactly the same! Each instrument gets its distinctive timbre from the multiple partials in its sound spectrum, and those arise from multiple simultaneous modes of vibration. This is explored a bit more in Chapter 201.

However, it is possible to cause a 1D instrument to vibrate in such a way that the lowest frequency mode is not active. In that case, the fundamental of the sound produced is not the fundamental mode of the standing wave medium. This chapter considers changing which mode of the 1D medium is the lowest *active* one, thus changing the pitch of the sound.

First, consider the cases where the ends of the medium are either fixed or free, so that the mode number (n or $m + \frac{1}{2}$) is restricted to positive whole numbers. In this case, the mode number is not useful for tuning. Tuning requires small adjustments in frequency, while changing modes causes large jumps in frequency. The discrete jumps in frequency caused by changing mode number do offer the potential to reach different pitches in a scale. Instruments vary widely in the use of this effect, mainly determined by how directly the musician controls the vibration source. If the musician is separated from the vibration (as with a piano or

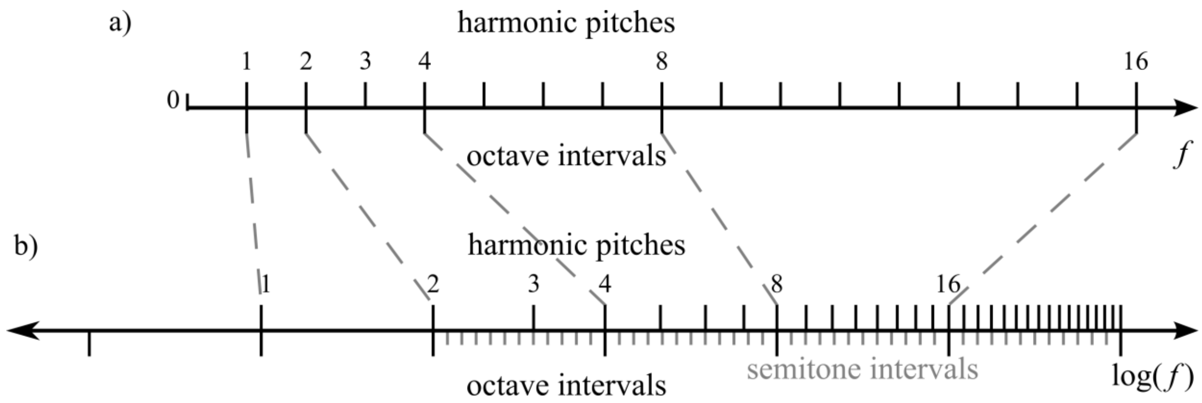

Figure 190.1

Relationships between harmonics and octaves, on (a) linear and (b) logarithmic scales.

pipe organ), mode control is not possible at all. For string instruments in general, mode control is not particularly easy, and is used more for its effect on timbre than on pitch (see Chapters 210 and 211).

For most wind instruments, the mouth is directly applied to the vibration source (although not with bagpipes, for instance). These generally use the lower harmonics for some pitch control, depending on the degree to which the player's **embouchure** (the positioning of the mouth) affects the standing wave. When the player's breath causes something else to vibrate, the first two or three harmonics might be used. For many brass instruments, on which the lips themselves vibrate (see Chapter 193), harmonics up to mode 8 are used (and as high as mode 12 for expert players). A few brass instruments use modes up through 16 and perhaps a bit higher; examples are the natural horn and its descendant the modern horn (commonly known in the United States as the French horn).

But for playing music, the utility of the mode number is limited by the frequency spacing of the modes. Harmonics are equally spaced on a linear frequency axis, while musical scales are equally spaced on a logarithmic frequency axis. The resulting relationship is shown in Figure 190.1, in the case where all the harmonics of the instrument's fundamental are available. Part (a) has a linear frequency axis, on which the octaves (arbitrarily starting from the first harmonic) get exponentially wider for higher frequencies. Part (b) has a logarithmic frequency axis, so that all octaves are the same width, and the harmonics get more and more crowded at higher frequencies.

Below roughly harmonic 16, the resulting musical intervals between the harmonics are too wide to provide a full chromatic scale. Above roughly harmonic 20, harmonic intervals quickly become significantly smaller than a semitone, which would make it hard for a musician to pick the right ones. Also, many of the harmonics 7 and above do not fall close to any desired pitch for the chromatic scale. There are some common instruments that work within these restrictions. For example, the bugle uses music limited to the five pitches of harmonics 2 through 6. But for a chromatic scale, another pitch adjustment mechanism is needed in addition.

So far, this chapter has worked within the restrictions of completely fixed or completely free ends. What about arranging for the ends of the medium to be between fixed and free, so that whatever disturbance makes the wave is possible but restricted. For string instruments, this is not really an option. To get sufficient tension in the string (providing the restoring force), fixed ends are required. For wind instruments, a partially fixed end can be created with an end that is partially open, that is, with a hole that is smaller than the tube cross section. This would allow n in Eq. 188.1 to differ from the integers or half-integers.

A difficulty with partially open ends is that the resulting series of mode frequencies is no longer harmonic. Although this chapter is focused on the production of pitches, the mode frequencies are also responsible for instrument timbre, which would be less pleasant with inharmonic modes. Slightly adjusting end openness is used on brass instruments. The modern horn and natural horn use this at the bell end to correct for the problem of higher modes not being at quite the right frequencies for the chromatic scale. In most brass instruments, something about degree of openness also occurs at the mouthpiece, although the sense in which that end is open or closed is complicated, as described in Chapter 198. But generally, departing from fixed or free ends is found to be impractical for playing scales, is not used for tuning (because there are easier options), and is only somewhat employed for instrument design (where it is more easily understood as adjusting the "effective length" of the instrument).

Chapter 191. Changing Length

The final available parameter for adjustment of standing wave frequency, the length of the medium, is the most useful of the three. It combines complete flexibility of pitches with ease of use. With very few exceptions, adjusting the length of the medium is the primary mechanism by which 1D standing wave instruments rapidly access different pitches to produce music. In addition, it is the only available parameter with which to tune wind instruments. Length is rarely used to tune strings, however.

For string instruments, lengths can be varied by having many different strings on one instrument, as on the harp, piano, and zither. One string can also be effectively shortened by temporarily fixing a point on the string, thus removing part of the string from the vibration. The shortened remaining string vibrates at a higher frequency. Examples include pressing with a finger on a guitar or violin string. A way to provide for easy and exact length adjustment is to add **frets**. These short bars perpendicular to the string, against which the string is pressed, appear on guitars and sitars, for example. These are not just markers indicating where to press the string. The musician presses the string just next to a fret, and where the string physically touches the fret makes the new fixed end that shortens the string.

For tubes of air, various methods of adjusting the length are detailed in Chapter 192. Many wind instruments have a tube that is not cylindrical in shape. The **bore** of the instrument, the long hole down the instrument's length, may be wider in some places than in others. As a result, although the three parameters of mode number, wave speed, and length continue to control the standing wave frequency, Eq. 188.1 may not be the correct relationship between them. But the proportion between frequency and length remains a good model even for these instruments,

$$f \propto \frac{1}{L} \quad . \qquad (191.1)$$

So as long as a question relates to different pitches from the same instrument, it is often possible to obtain answers based only on this proportion. It is of course necessary for all other circumstances to remain constant. But in cases where it applies, using this can circumvent any need to know other details about the instrument.

Chapter 192. Length Control for Tubes

There are at least four different ways commonly used to adjust the effective length wind instruments for the purpose of making the notes on a scale.

The mechanically least sophisticated option is to not change the length at all, but instead to provide a set of tubes with a variety of fixed lengths. Pan flutes and pipe organs have one pipe or tube for each pitch. This is the same strategy as used for the strings in a harp or piano. In fact, collections of tubes have one advantage over collections of strings. Strings need to be tuned frequently, and this becomes a major task for a typical piano with 215 strings. But once the lengths of a set of tubes has been correctly set, they rarely need adjustment in order to stay in tune.

A second option is to provide a **slide** that shortens or lengthens the tube. In a slide whistle, the closed foot can move up the tube, shortening it. The slide of a trombone similarly extends the tube when pulled out, and by being in the middle of the tube has the added advantage of not limiting whether the ends are closed or open. A potential down side for this solution is that it takes considerable practice to know where to position the slide for specific pitches. However, this is hardly insurmountable (and is an aspect of many string instruments as well). A more significant restriction with slides is that they require the tube to be cylindrical; as covered in later chapters, many instruments actually have tubes with diameters that change along their length.

The most mechanically complex option is to add **valves.** Valves exploit the fact that, unlike strings, there is no need for the tube of a wind instrument to be straight. Just as a fluid can flow through a tube with twists and turns, a standing compression wave also can occur along the length of a curved tube. Figure 192.1 shows the basic idea. The dashed circle is the valve, with a wall across the center. The valve can rotate by 90°. When the wall is horizontal (light gray in the figure), the tube is open straight from left to right. When the valve is rotated to make the wall vertical (black in the figure), the path through the tube has to take a detour through the loop below (as represented by the dotted line). Valves change the tube length by a specific, predetermined amount. Valves are almost always arranged so that when activated they lengthen the tube, lowering the pitch as a result.

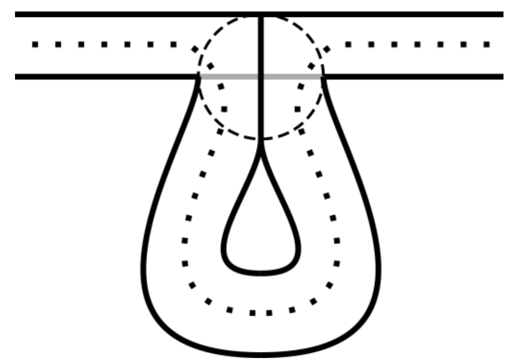

Figure 192.1

Schematic of a rotary valve action. Valve is dashed circle with wall (black line). As shown, the loop below is added to the total length of the tube. When rotated to put wall on gray line, loop is excluded.

Valves come in various shapes. The one represented in Figure 192.1 is a **rotary valve**, commonly found on horns and tubas. Trumpets more often have **piston valves**, which move up and down. The details vary, but for the purposes of this book the only important part is that valves add length to tubes.

The final common length adjustment method is the addition of **tone holes** along the length of the tube. Their use is somewhat similar to fixing a point on a string, because this method removes part of the tube from participating in the standing wave, thus shortening the active length and raising the pitch. Figure 192.2 shows a tube that is open at both ends, and tone holes that are either closed (filled circles) or open. The figure also superimposes on the pictures a graph of the density/pressure standing wave of the fundamental mode. The mouthpiece must be on the left end of these tubes, so that it can create the standing waves.

In part (a), the tone holes are closed, and therefore effectively not present, resulting in a standing wave that has a density/pressure node at each end of the tube. In Figure 192.2(b), tone hole (i, the right-most one) has been opened. As with the open ends of the tube, this opening allows the constant density of the air outside to hold the inside constant, moving the node to the tone hole. The portion of the tube between the tone hole and the right end now has a constant air density throughout, and is removed from the standing wave.

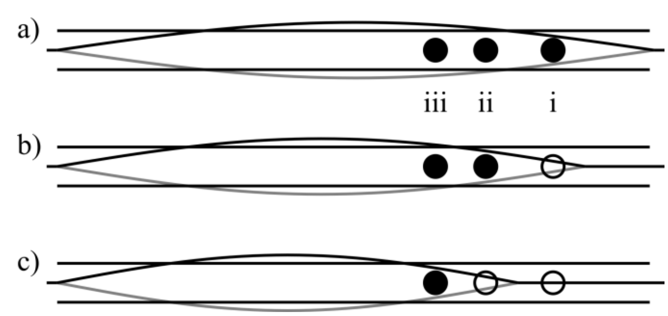

Figure 192.2

Tube with tone holes. Black holes are closed, white holes are open. Dashed lines show pressure graphs of fundamental standing wave.

Except, it is not quite like that. Tone holes must have a diameter that is smaller than the tube diameter, and often they are significantly smaller. This means that they only *partially* open the tube to the air, with the result that the density and pressure variation at the tone hole is reduced, but not totally eliminated. The standing wave end node is shifted in towards the tone hole, but not exactly to it.

When the next tone hole is opened, (ii) in Figure 192.2(c), the node moves inwards even further. In the simplest model, tone hole (i) has now become superfluous, since tone hole (ii) and the right end of the tube are now holding the entire portion between them at a constant pressure. However, in practice, and partly due to the effect described in the preceding paragraph, the musician leaves all tone holes open, from the tube end to the new node position. The basic method is to progressively open holes, from the foot inward, to get a sequence of pitches.

Mechanically, tone holes may simply be holes covered with the fingers, as on the recorder and tin whistle, or may be covered by mechanical **keys**, as on the modern flute, clarinet, or oboe. Keys are especially needed when either the size or spacing of the holes is too large for fingers, as on the saxophone. Keys are sometimes designed so that the hole remains covered until the musician presses the key. This can lead to major confusion in terminology: sometimes a *key* is described as “closed” when it is depressed, which may mean that the *hole* is opened.

Chapter 193. Wind Instrument Vibration Sources

Although the construction of musical instruments defines which frequencies they can produce, they do not play themselves! Energy must be provided, usually by the musician, and certainly under the control of the musician. Sometimes that energy is oscillatory, but often it is not. For wind instruments, there are three primary mouthpiece types which deliver the energy, and these define three major instrument categories.

1. Buzzing lips are the source for **brass** instruments. The lips are held tightly together, and then breath is forced through them such that it escapes in extremely rapid pulses. Doing this is greatly assisted by a cup-shaped mouthpiece on the instrument, but it is possible to do without the instrument at all.

2. **Reed** wind instruments have a flat, flexible but stiff reed, usually in a long rectangular shape, which is attached to the instrument so as to leave a small gap into the tube interior. For **single reed** instruments, the gap is between the end of the reed and the tube body. For **double reed** instruments, two reeds are placed together, with the small opening between the two reed ends. The musician provides a steady stream of air, which has to pass through the gap to get into the tube. The extra pressure from the musician’s blowing tends to bend the reed(s) to close the opening, preventing this airflow. But if started correctly, the reed(s) will oscillate, allowing puffs of air into the tube.

   Reeds in these instruments operate in a different way from the reeds of the thumb piano or harmonica. In those cases, the natural frequency of the reed itself determines the pitch. For reed wind instruments, the reed provides the sound energy, but must follow the frequencies from the tube of air, as described in Chapter 197.

3. Oscillating flow of air itself drives the **flute** category. Here, the musician blows a steady stream of air at the edge of a hole. Sometimes there is a small windway that directs the stream of air at the edge; this whole structure is a **fipple,** and the hole next to the edge is the **fipple hole.** Otherwise, it is up to the musician to aim the air properly; in this case the hole is called the **embouchure hole.** You might expect the air stream to simply split in two, with some flowing to each side of the edge. This would lead to some air entering the instrument body, and some staying outside the instrument. However, for reasons that are described in Chapter 196, the air stream instead alternates rapidly between going over and under the edge. This is called an **air reed.**

The flutes and reeds are collectively called **woodwinds.**

The terms in this taxonomy seem as though they were intentionally chosen to mislead.

- Brass and woodwind sound as if they describe the construction material. But while most modern brass instruments are made of metal, there are historical brass instruments that were normally made of wood. Some woodwinds are also made of metal, most prominently the saxophone and concert flute, and others are commonly made of plastic. In any case, making a specialty version out of an unusual material does not change an instrument's classification. Material choice can have a subtle effect on the timbre of a wind instrument, but such effects are not discussed in this book.
- Reed instruments are so named because originally reeds came from specific woody plants. For many instruments that is still the best or only source. But some reed instruments may have reeds made of plastic or metal.
- Many pipe organs have so-called reed pipes. However, they do not function in the same way as woodwind reeds. In fact, although these pipes are made to sound by blowing air, they are not strictly wind instruments, because the pitch is not controlled only by standing waves in a tube.
- The word flute may refer to the entire third category, but it may also refer specifically to the common instrument that is just one example of that category. In this book, the specific instrument will be referred to as the concert flute.
- An air reed has almost nothing in common with the reed in a reed instrument, other than the fact that they both oscillate.

Chapter 194. Instrument Response Curves

194a. Response Curves of 1D Medium

When a musician continuously puts energy into an instrument, they **drive** the oscillation of the standing wave. Not all instruments call for this, but many do. How will the instrument respond? Chapter 89 describes how to characterize the response of a **driven oscillator** with a **response curve**. The same idea can be applied here: vibrate the standing wave medium sinusoidally at many different frequencies and measure the size of the resulting standing wave.

The input can be anything that will disturb our one-dimensional medium at a specific frequency. For instance, it could be a way to sinusoidally vibrate the end of a string, so that the input amplitude can be measured. Or it could be a small speaker applied to the end of a wind instrument, where the acoustic power delivered into the tube could be measured. The output of the system might be the sound power coming out of the wind instrument, which would fit well with the ideas in Chapter 89. Or, the resulting vibration might be more easily measured by the amplitude of the resulting standing wave, in which case the results of the experiment would be more like the amplitude graph in Chapter 68.

In any case, the qualitative result would be the same. In both of Chapters 68 and 89, we saw that if the system being tested has a natural frequency, then the response is especially large when the driving frequency matches that natural frequency. This is the phenomenon of **resonance.** A 1D medium has multiple natural frequencies, arising from the modes described in Chapters 179 and 181. Near each of these natural frequencies there will be a resonance in the response. So, unlike the single-peaked response curves from earlier chapters, the response curve will have multiple peaks, as in Figure 194.1(b).

In real experiments on wind instruments, rather than wave amplitude or power gain, results are often given as the instrument's **acoustic impedance** versus driving frequency, called an **impedance curve.** As when impedance appeared in Chapter 149, this book will not go into what it means. It is only mentioned so that if you encounter impedance curves in other references, you can think of them as response curves.

194b. Response Curves and Sound Production

As with the resonant cavities in Chapter 96, resonances of a wind instrument's response curve indicate those frequencies that will be emphasized in the sound coming out of an instrument, regardless of how that instrument is driven. Can this account for the fact that these instruments produce only specific pitches?

If you listen to background noise through a tube of around 1 m long, you can clearly hear the effect that Figure 194.1(b) represents. All sounds acquire a tonality that is determined by the fundamental resonance frequency of the tube. This is because the input frequencies near those harmonics are emphasized, and others are de-emphasized. Similarly, if you speak through such a tube, the resonant frequencies in your voice are enhanced. However, the effect is not so strong as to turn your speaking into musical notes! It turns out that the resonances of wind instruments do not have a particularly high quality factor Q, so the response curve peaks are rather broad. Although input frequencies between the resonance peaks are de-emphasized, they are certainly not eliminated.

Thus, these response curves show which frequencies a wind instrument prefers, but they don't explain why the instrument so strongly adheres to those frequencies. There is some missing ingredient for this explanation, which is covered starting in Chapter 195.

As described in Chapter 99, a moderate Q value is linked to the fact that energy leaves the system quite quickly. When a musician stops playing a wind instrument, the sound stops almost immediately. This is why these instruments must be continuously driven while they are being played. The same general principles apply to string instruments which only make sound while being played, such as the violin. This is as opposed to a guitar, with sounds that sustain after the musician has stopped action; this discussion does not apply to them.

Chapter 195. Feedback

195a. Instrument in Charge

For some string instruments, the wave medium is deflected from equilibrium and then suddenly released. This allows it to vibrate on its own. An example is plucking a guitar string. The act of starting the sound is separated from the time when the vibration happens.

But for other stringed instruments, and all wind instruments, the cause of vibration is applied while the wave is vibrating. The bowing of a violin, or the blowing on a reed instrument are examples. In these cases, the musician supplies a steady action, not an oscillating one. Compare this to starting with a mass

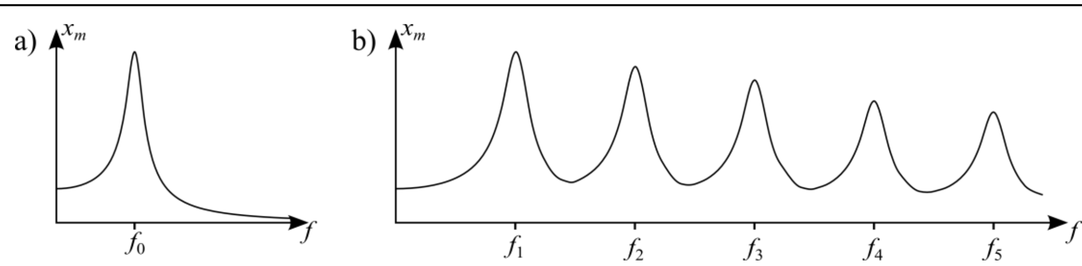

Figure 194.1

Graphs of amplitude versus driving frequency. (a) Mass on spring (duplicate of Figure 68.1). (b) A medium that can support a one-dimensional standing wave.

hanging from a spring, and then applying a steady force to it; the spring will stretch to a new equilibrium and then remain stationary until released. How can a steady disturbance result in oscillatory behavior?

A similar issue even applies to brass instruments driven by buzzing lips. The vibrating lips are oscillating, with their frequency controlled by the musician. But the pitch of the instrument is supposed to be determined by the body of the instrument. How does the musician find just exactly the frequency that the instrument is set to? How does the violin bow, dragged steadily across the string, cause just the right frequency?

The resolution of these questions is provided by **feedback.** Feedback refers to any situation where the chain of cause and effect circles back on itself. Often it takes the form of two devices, each of which affects how the other behaves. The idea of feedback shows up in an enormously wide range of contexts. Feedback is different from the idea of response (described starting with Chapter 89), in which the output of a device or system depends only on the input provided. When the output circles back to affect the input, the response concept is no longer sufficient to understand the system.

People are often familiar with an unpleasant example of acoustic feedback: the squealing that is created when a microphone gets too close to a speaker. Any sound that is picked up by the microphone is turned into an electrical signal, which is amplified and then sent to the speaker. The speaker turns the amplified electrical signal into a sound, which then travels through the air and can be picked up by the microphone, thus completing a loop of cause and effect. The result can be a piercingly loud tone.

In order for this to happen, when a sound makes one trip around the loop, it must increase in power. If this is the case, then any very small sound can grow louder and louder. If not, then the sound will die away as it runs around the loop multiple times. This is the reason that the feedback problem only occurs when the microphone is close to the speaker. Only a fraction of the sound leaving the speaker is picked up by the microphone, with the rest of the energy lost to the loop. If the amplification is not sufficient to make up for that loss, then the sound will get smaller and disappear. Moving the microphone closer to the speaker increases the fraction of the speaker sound that is captured by the microphone, which can push the situation into the other mode.

To make sound with a 1D musical instrument, the musician must supply a driving force, which pumps energy into a standing wave in the instrument medium. Much of the energy then leaves rather quickly, creating sound. But some of that standing wave energy must feed back to the musician, in a way that ensures that the driving force vibrates at the frequency for which the instrument is set. In some cases, that feedback even helps the driving force to be oscillatory instead of steady. The details vary by instrument and are reviewed case-by-case in Chapters 196–198 and 209.

Chapters 196–198 start with a model which is unrealistic in one respect. In 1D musical instruments, the ultimate result is a standing wave with quite a long wavelength. However, it is easier to understand the feedback mechanism if we instead imagine very short pulses of compression traveling through the medium. This eases consideration of how the pulses reflect off the ends of the medium. To reconstruct realistic standing waves, we must imagine widening these pulses, to the point that they resemble the countermoving waves of Figure 177.1. The superposition of the left-going and right-going parts form the standing waves of Table 179.1 or Table 181.1, as appropriate.

195b. Extra: Feedback Timing

In addition to a loop that increases to strength of a sound signal, there is a second criteria to be considered for feedback loops. Because sound is an oscillatory effect, it is important whether the sound is nearly in phase with itself after one loop.

Consider the microphone-and-speaker system in Figure 195.1. Suppose that the time required for one trip around the loop is Δt_c (c for cycle), and imagine starting the process with some soft background sound into the microphone with period $T = 2\,\Delta t_c$. In Figure 195.1(a), a maximum of the sound pressure is just reaching the microphone. In Figure 195.1(b), half a period later, the sound pressure minimum is reaching the microphone just as the maximum, having traveled through a feedback cycle, is also reaching the microphone. Because they are out of phase, the two will tend to cancel.

When a trip around the loop leads to destructive interference, as in Figure 195.1, it is called **negative feedback.** This might sound "bad," being associated with destruction. But a better way to think of negative feedback is as a stabilizing force, preventing deviations from equilibrium. Used in that way, negative feedback is an essential component in the design of many electronic circuits.

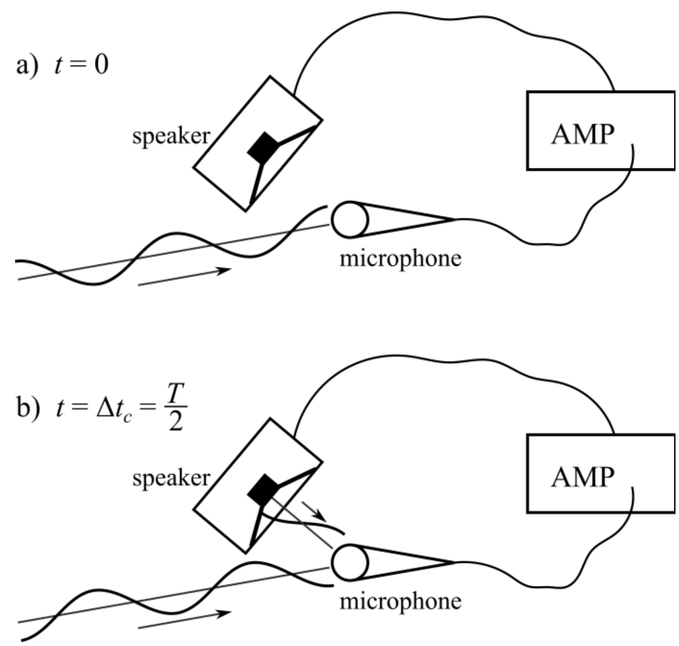

Figure 195.1
Feedback of a sound that ends up out of phase with itself.

When a trip around the loop leads to constructive interference, it is called **positive feedback.** For example, if the input period in Figure 195.1 were equal to the loop delay, $T = \Delta t_c$, then the two signals would reach the microphone in phase. Positive feedback is not always good, because it can result in run-away behavior. The sound could continually reinforce itself, getting louder and louder. This timing requirement is the reason that the feedback results in a particular tone. Only sounds with the right period are successfully amplified.

Positive feedback is exactly what is needed to create a standing wave in the medium of a 1D musical instrument. This ensures that the musician is synchronized with the standing wave vibration, so that energy flows from musician to instrument.

Chapter 196. Air Reed Feedback

Consider the feedback in a concert flute, with an embouchure hole at the mouthpiece and an open foot. In Figure 196.1(a) a cycle starts with air being blown into the embouchure hole. This has two consequences. It creates an overpressure pulse inside the tube, which (b) travels away from the embouchure hole, while the overpressure (c) pushes the airstream to no longer enter the embouchure hole. When the overpressure pulse (d) reaches the far end, it reflects with inversion, since an open end has a fixed pressure. This results (e) in an underpressure pulse, which returns (f) to the embouchure hole. Finally, (g) the underpressure sucks the airstream back into the embouchure hole, which then fills the tube with an overpressure pulse, and the cycle starts over again.

The overall result is that the action inside the tube feeds back to determine the frequency with which the airstream drives the instrument. At the mouthpiece, notice that the overall effect is to reflect the pulse with an inversion. Although the details just described involve more than just reflection, that end is effectively an open end, making the flute as a whole effectively an open tube.

Notice also that in the course of one cycle, the pulse travels a distance $2L$. This matches the denominator of Eq. 179.1, which gives the fundamental frequency for an open tube. This match is not a coincidence. If the pulses were widened to be as long as the tube, their superposition would be precisely the fundamental standing wave of Table 179.1.

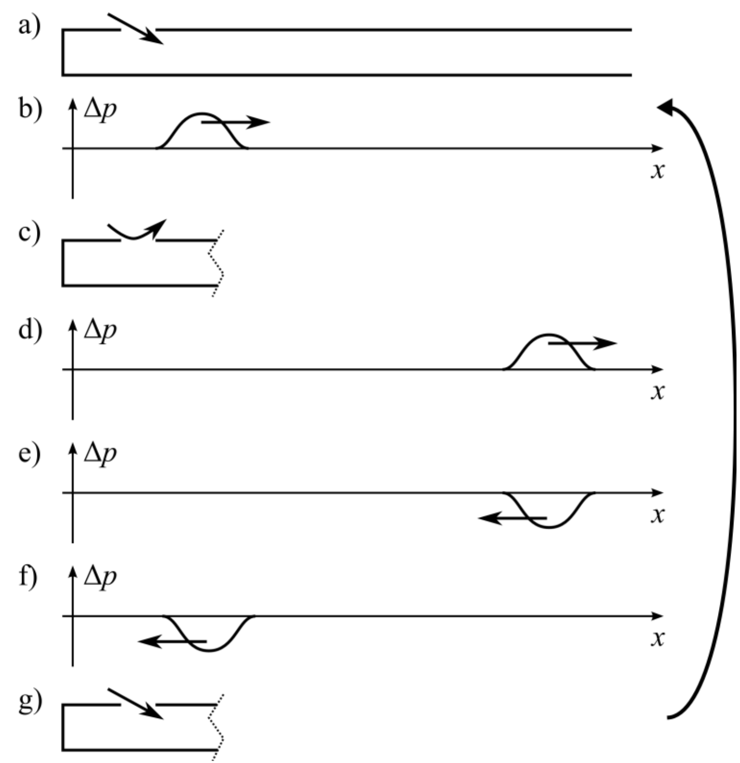

Figure 196.1

Feedback cycle for open-ended flute.

Similar cycles are possible for the higher numbered modes. The second harmonic mode, with double the frequency, would call for the airstream to oscillate twice as fast, so that it would be blowing in to the tube just after time (d), halfway between (a) and (f). That would launch a second overpressure pulse from the embouchure hole. In line with the principles of superposition, we can then think of those two pulses as independently traveling up and down the tube.

A comparison of Figure 196.1 and the bowed string feedback in Figure 209.1 is worth noting. The driving mechanism is very different, but both involve pulses traveling along the medium and inverting at the ends. The two figures do not start at the same point in the cycle; can you figure out how they match up?

Chapter 197. Reed Feedback

Consider the feedback in a clarinet, with a reed mouthpiece and an open bell. In Figure 197.1(a) a cycle starts with air being blown into the tube through the reed gap. This has two results. It creates an overpressure pulse inside the tube, which (b) travels away from the reed. The blowing pressure outside of the reed also starts to bend the reed up, towards closing the gap. When the overpressure pulse (c) reaches the far end, it reflects with inversion, since an open end has a fixed pressure. This results (d) in an underpressure pulse, which returns (e) to the reed end. The underpressure sucks the reed upwards, to complete the process (f) of closing the gap.

Since the closed reed gap prevents any breath from entering the tube, the underpressure pulse makes an upright reflection off the closed end, without inversion. The underpressure (g) travels down to the open end, where it (h) once again inverts and returns to the reed. This time the overpressure helps to push the reed open, allowing the air back in through the gap. Air is blown in to reinforce the overpressure, and the cycle starts over again.

The overall result is that the action inside the tube feeds back to determine the frequency with which the reed vibrates and air enters the instrument. At the mouthpiece, notice that the overall effect is to reflect the pulse without inversion, at both time (f) and time (i). Although the details just described involve more than just reflection, that end is effectively a closed end, making the clarinet as a whole a closed tube.

Notice also that in the course of one cycle, the pulse travels a distance $4L$. This matches the denominator of Eq. 181.2, which gives the fundamental frequency for a closed tube. This match is not a coincidence. If the pulses were widened to be twice as long as the tube, their superposition would be precisely the fundamental standing wave of Table 181.1.

Similar cycles are possible for the higher numbered modes. A mode with double the frequency would call for the reed to oscillate twice as fast, so that the gap would be open at (f), halfway between (a) and (i). But that would result in negative feedback, with air through the hole tending to fill the underpressure and squelch the pattern. Therefore, in agreement with the closed tube formula, the next highest mode will be the 3rd harmonic, with the reed gap opening three times during the cycle illustrated, creating three pulses. In line with the principles of superposition, we can think of those three pulses as independently traveling up and down the tube. Three pulses each making two round trips per cycle sounds like quite a mess, but it is one way to construct the third harmonic mode.

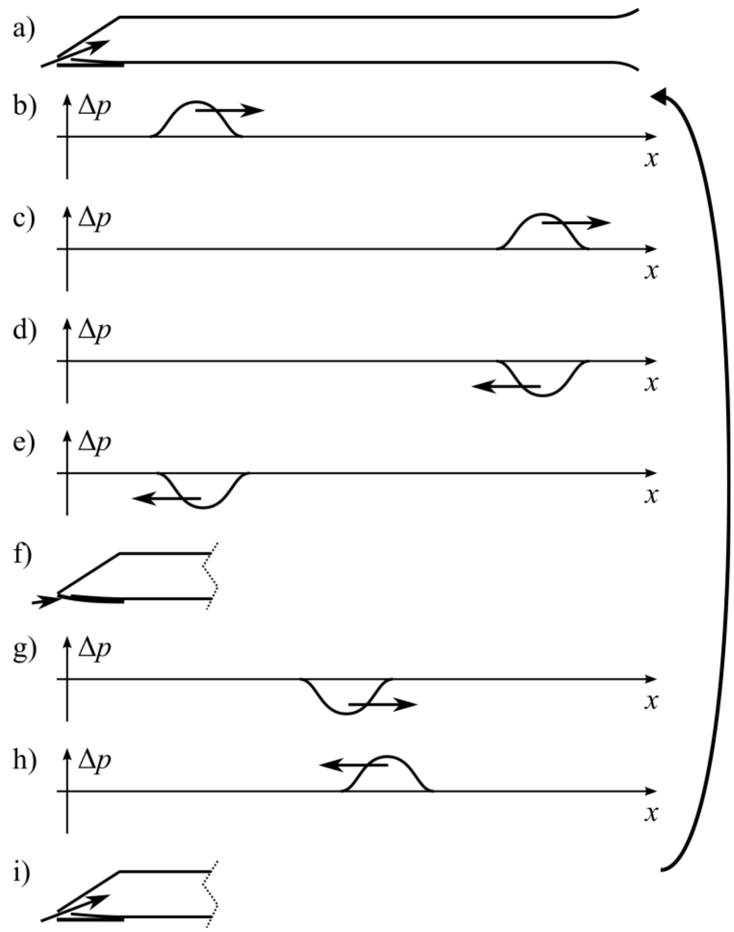

Figure 197.1

Feedback cycle for open-ended reed instrument.

Chapter 198. Lip Feedback

How best to model brass instruments is still an area of active investigation. It appears that a really good model for the driving force must be more complex than just one object subject to one restoring force.[62] The following simpler model gives insight to the basic wave behavior. But since it is incomplete, be warned that some of the described cause and effect may not make incontrovertible sense.

In Figure 198.1(a) a cycle starts with air being blown into the mouthpiece through parted lips. This launches an overpressure pulse inside the tube, which (b) travels away from the lips. When the overpressure pulse reaches the open bell of the instrument, it reflects with inversion, since an open end has a fixed pressure. This results (c) in an underpressure pulse, which returns to the lip end.

The player has adjusted their embouchure so that the lips vibrate at roughly the desired frequency. As a result, the underpressure pulse returns to find (d) the lips closed. The underpressure, along with increased pressure inside the mouth, result in a force pulling the lips back into the mouthpiece, which would open

[62] Henri Boutin, Neville Fletcher, John Smith, and Joe Wolfe, "Relationships Between Pressure, Flow, Lip Motion, and Upstream and Downstream Impedances for the Trombone," *J. Acoust. Soc. Am.* 137(3) (2015): 1206–1208.

them. However, the mass of the lips is large enough that they accelerate comparatively slowly, and the underpressure pulse makes an upright reflection off the closed end before the lips actually open.

The underpressure (e) travels down to the open end, where it (f) once again inverts and returns to the lips. By this time, the lips have opened fully. Air is blown in to reinforce the overpressure, and the cycle starts over again.

The brass player is a more active participant in this process than for the woodwinds. To be clear, the player is not actively opening and closing the lips, as it would be impossible to do so rapidly enough. Rather, the lip tension and shape are controlled so that the lips have a natural vibration frequency. However, feedback is still in operation, as the returning pulses, especially the underpressure at step (d), pull the lip vibration frequency towards an instrument resonant frequency. At the mouthpiece, notice that the overall effect is to reflect the pulse without inversion, at both time (d) and (g). Although the details just described involve more than just reflection, that end is effectively a closed end, making the brass instrument as a whole a closed tube.

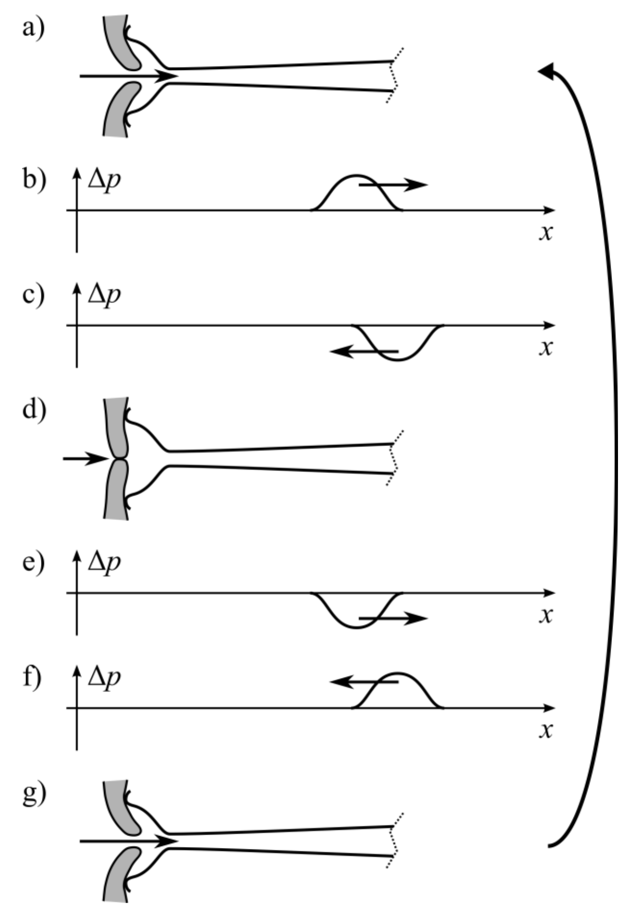

Figure 198.1

Feedback cycle for open-ended brass instrument. Gray shapes represent players lips, pressed against a brass mouthpiece.

The more active role of the player enables brass instruments to more extensively use multiple vibrational modes than other instruments. Tighter lips lead to the lips opening multiple times during the sequence in Figure 198.1. In line with the principles of superposition, we can think of those pulses as independently traveling up and down the tube, although their superposition creates a standing wave.

The control exerted by the player also allows for a technique called **lipping,** by which the vibration frequency is forced to be as much as a semitone different from the tube resonance frequency. It is therefore less crucial that the instrument's resonances match the frequencies of the desired musical scale, because discrepancies can be corrected by lipping.

Chapter 199. Tube Ends and Flared Tubes

When a tube contains a standing compression wave, the length of the standing wave may not be precisely the same as the physical tube length. The length of that wave is called the effective length, or the **acoustic length**, of the tube. You may have read Chapter 184, which describes how for a simple tube with an open end, the standing wave extends a bit beyond the end.

In principle this effect applies to wind instruments as well, but in most cases it is difficult to define with precision. Sometimes, as with various mouthpieces, it is not clear precisely where the "end" of the tube is located. If it isn't clear exactly what the physical tube length is, then it makes little sense to speak of a correction to that length. In other cases, the tube end has some shape (such as a bell) which has a clear ending position, but which nevertheless clouds its relationship to the model of a cylindrical tube. In practice, it can be easier to find the acoustic length of an instrument through sound, by measuring the normal mode frequencies and applying the appropriate equation.

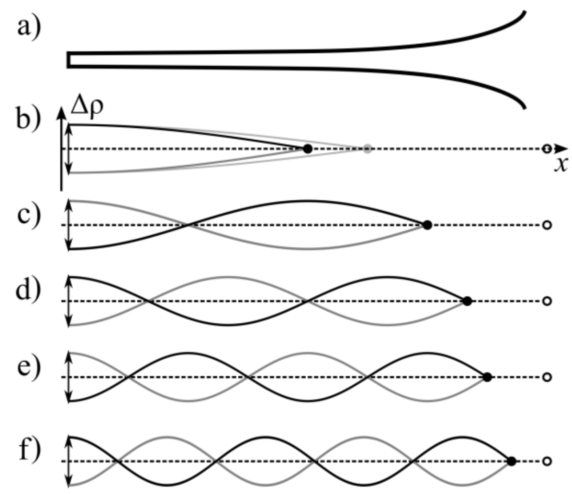

Figure 199.1

(b–f) Standing waves for closed tubes, lengths adjusted to give frequency series of a brass instrument. (a) How a flaring bell would be oriented to achieve this. (c–f) White circles show length of open tube with same mode frequencies. (b) Gray shows length required for fundamental of this harmonic series.

The brass category includes large variety of shapes, all of them quite different from a cylindrical tube. Almost all have a bell that flares out significantly. Some brass instruments are essentially conical up to the bell, such as the tuba, baritone, and cornet. Others combine conical sections with cylindrical sections ranging roughly from one-third (trumpet) to two-thirds (trombone) of their total length. At the other end, even the shape of the mouthpiece, small as it is, has important effects on the standing waves in the tube. These shapes have multiple consequences, but perhaps the most important is to change the lengths of the standing wave normal modes. Even though the mouthpiece acts as a closed end, the normal mode frequencies are *not* those of a closed tube.

The part of this length modification that's easiest to describe is how the flared bell creates an end correction that depends on frequency. The terminal pressure node of a standing wave at an open end occurs at the point where a traveling wave would reflect. Chapter 148 describes how compression waves reflect from abrupt changes in the medium. But what, exactly, does "abrupt" mean? This sort of question arises frequently in physics. A physical model is often accurate as long as some parameter is "small" or "large," but sometimes even experienced physicists forget to answer the question, "Compared to what?" One natural tendency is to compare to the usual range of human experience: an elephant is "large" and an insect is "small." But nature is not really anthropocentric, so in physics there is always a better answer.

In this case, reflections occur when the 1D medium changes rapidly compared to one wavelength. This means that a change in tube diameter which takes place over 5 cm of length might be abrupt for a wave with a wavelength of 1 m, but would not be abrupt for a wavelength of 0.5 cm. The flare of a brass instrument bell gets more and more extreme towards the open end. For long wavelength modes the "end" of the tube is deep in the throat of the bell, where the flare is comparatively gradual. Shorter wavelength modes reach further towards the end of the instrument, to a point where the flare is widening more rapidly. The resulting end correction is very different from the one described in Chapter 184. It has the opposite sign, with the acoustic length shorter than the physical length. Higher frequencies give a longer acoustic length, the opposite of the Chapter 184 case, and this end correction changes more rapidly with frequency.

The specific, characteristic shape of a brass instrument bell is chosen to harness this effect, with the intent of making the mode frequencies include all harmonics of some fundamental. To give an idea of this goal, Figure 199.1 shows a series of normal modes for closed cylinders, with lengths adjusted so that their frequencies are all harmonics of a fundamental (part b, gray). It is apparent that to achieve this goal, the effective length for the fundamental would have to be far less than the physical length, roughly half the length. However, the needed end correction is much less extreme even for the second mode, and the correction continues to get smaller for the higher modes.

The bell flare is only part of the story. Other elements of the tube shape, especially the mouthpiece on the end (which can be modeled as a Helmholtz resonator), all work together to modify the normal mode

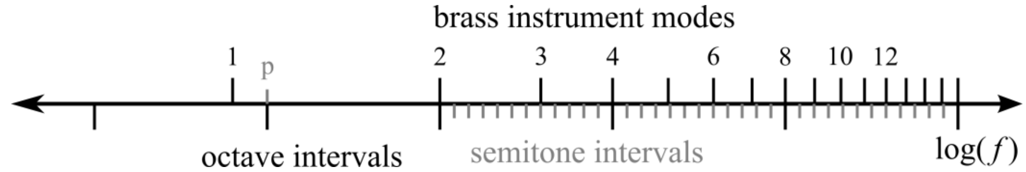

Figure 199.2

Frequencies of modes for typical brass instrument, labeled by mode number, which is also the harmonic number for all except the fundamental. Harmonics are more crowded on right due to logarithmic frequency axis.

frequencies. The final design is crafted so that modes two and higher, at least as high as 12, form a complete harmonic series with the harmonic number matching the mode number.

$$f_n = nf_p \quad , \quad n = 2, 3, \dots \quad . \tag{199.1}$$

These are illustrated on a logarithmic axis in Figure 199.2. The base of this harmonic series is called the **pedal tone**, labeled with a gray 'p' in Figure 199.2. Due to the adjustments in tube shape, the pedal tone is approximately related to the instrument length by

$$f_p = \frac{s}{2.1L} \quad . \tag{199.2}$$

While this is quite close to Eq. 179.2 for an open-ended tube, the exact multiplier of 2.1 is not a rule of physics. In fact, that multiplier is slightly different for different models of the same instrument.

Eq. 199.1 does not include $n = 1$ because the tube shape modifications do not succeed in modifying the frequency of the first mode to match the rest of the harmonic series. Considering in Figure 199.1(b) how short the required effective length is, it is no surprise that it is not achieved. The real first normal mode, a standing wave with only one pressure node, has a frequency roughly given by using $n = 0.8$ in Eq. 199.1. This is the frequency labeled '1' in Figure 199.2, and the corresponding closed-tube mode is Figure 199.1(b, black). This mode is not musically useful, and is rarely played unless further modifications are made as described below.

It is possible for brass players to play a note at the pedal tone frequency, although it is quite difficult to make it musically pleasing. When the pedal tone is played, it is actually the second and third modes that are primarily active. See Chapter 74 for why this would sound like a lower pitch.

The effective length of a brass instrument often involves even more complexity, in the form of **tuning compensation.** For instance, the length of extra tubing that is added by valves is often designed to be adjusted slightly during performance, rather like a mini-trombone slide that only takes effect when that valve is depressed. Tubas and euphoniums have additional compensation plumbing, activated by an additional valve, which fixes the frequency of the lowest mode so that they can reach the lowest notes possible. And horn players take matters into their own hands, literally, by placing a hand in the bell of the instrument. By adjusting the position of the hand, the player changes the degree to which that end is open, with the effect of adjusting the effective length of the tube.

Chapter 200. Brass Modes and Valves

Brass instruments rely more on changing modes than the woodwinds do. This may be more a result of evolution than of physics, due to the fact that brass instruments originated from horns on which mode changing was the only way to change pitch. But there are several ways in which this behavior fits well with the limitations imposed by physics.

Chapter 199 mentions that the lowest normal mode on brass instruments has a frequency that does not fit in the harmonic series defined by the other modes. This means that the lowest playable pitches used are based on the second mode. In Figure 190.1, while woodwinds start at mode 1, the brass start at mode 2, closer to the frequency range where changing modes becomes useful.

The top row of Figure 200.1 shows the harmonic normal mode frequencies relative to the chromatic scale. A basic brass instrument has three valves, which add enough tubing to drop the pitch by either one semitone, two semitones, or three semitones. Traditionally, those are respectively the second, first, and third valves. The figure shows how applying those valves to each of the mode frequencies results in an array of possible pitches.

Unlike tone holes, which are opened progressively from the foot, the valves on a modern brass instrument operate independently. Pressing two valves together drops the pitch by *almost* the sum of the individual valves. (Not quite enough, as you could calculate, but close enough for the performer to use.) So, all possible combinations of the valves can produce seven different notes. Seven just happens to be the number of semitones between the second and third harmonic modes (see Figure 200.1). If the modes had been more widely spaced, three valves would not have been enough to reach all the desired pitches.

Most brass instruments use from the second to the twelfth modes, with notes requiring modes above 8 considered to be very high. The 7th and 11th modes are avoided, as they are not naturally near the desired pitches (regardless of which valves are pressed). Because higher modes get crowded on a logarithmic frequency scale, for the higher pitches there are several **alternate fingerings,** indicated by the thinner marks in Figure 200.1.

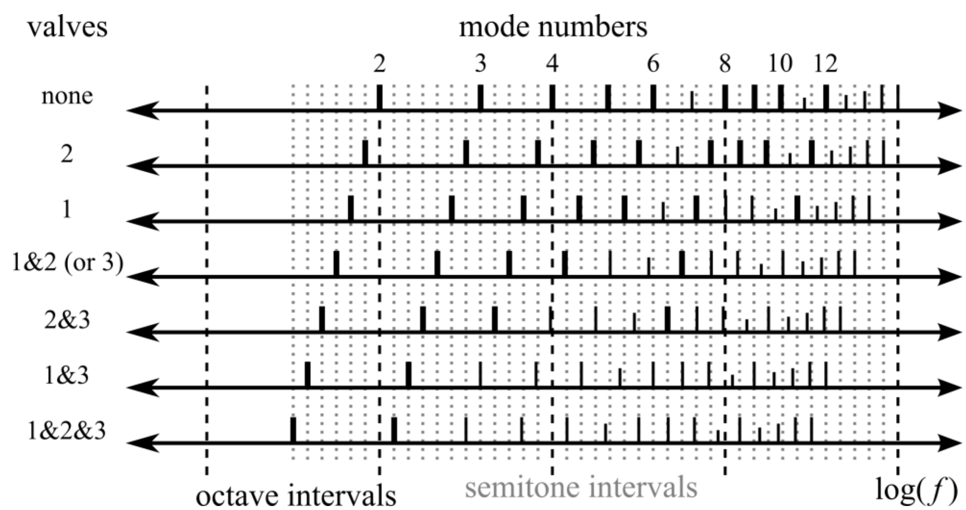

Figure 200.1

Pitches available on a basic brass instrument. Each row is one valve combination, assumed to lower pitch by an exact number of semitones. Marks for primary fingerings are thicker. Marks that aren't near notes of the scale are shorter.

The frequency ratio from the 11th to the 12th mode is small, between a semitone and a whole tone:

$$\frac{12}{11} = 1.090909 \quad , \quad R_s^2 = 1.05946^2 = 1.12246 = R_w \quad . \tag{200.1}$$

With the modes so closely spaced in frequency, not many pitches are obtained from any one mode. Therefore for upper brass pitches, the concept of "a set of pitches all played on the same mode" is not terribly useful. For this reason, the word **register** is often used for brass instruments to simply mean a range of pitches, with no connection to any particular mode.

Chapter 201. Timbre and Multiple Modes

When an instrument is played, it is highly unlikely that it will vibrate in only one of its normal modes. Common driving forces simply are not sinusoidal. Also, feedback can reinforce all available modes in the standing wave medium, not just the fundamental. For instance, in terms of the feedback models described in Chapters 196–198, feedback of modes higher than the fundamental occurs by having multiple pressure pulses circulating up and down the tube simultaneously. Thus, the standing wave that results will be a superposition of several, and often many, normal modes.

Each active normal mode in the instrument will contribute to the sound of the instrument. Each normal mode has its own frequency, which associates that mode with a partial in the sound spectrum. The relative amplitudes of the various normal modes will control the amplitudes of the partials, and will therefore control the timbre of the instrument. This book will not address how specific instrument characteristics can shape the timbre. For some instruments, playing techniques can adjust the standing wave amplitudes relative to each other, thus adjusting the amplitudes of the sound partials and the resulting timbre.

Variations in the relative amplitudes of the normal modes will not, however, have any effect on the pitch of the instrument. The frequency of a harmonic sound is generally equal to the frequency of its fundamental (recall Chapter 44). This is why, in most cases, in order to relate the pitch of an instrument to its length and other characteristics, we need only consider the fundamental frequency of the sound, which may or may not come from the fundamental vibration mode of the instrument.

Chapter 202. Bore Shape

Up to this point, when this book discussed tubes, it has focused on cylindrical ones: tubes with a circular cross section of the same diameter all along the length. The tube that is important for wind instruments is the **bore,** or the *interior* of the instrument. The outer shape of a wind instrument does not always match the shape of the bore.

Nearly all common wind instruments have circular cross sections. But since the waves are longitudinal, the cross-sectional shape is of little consequence. The medium moves along the tube, a motion that can occur equally well whether the bore cross section is a circle or a rectangle or anything else. A circular hole is simply the easiest shape to manufacture.

Of more significance than the shape is the cross-sectional area of the bore. In many common wind instruments, this area varies along the instrument's length. This does influence the waves, because moving some air from a wider to a narrower part of the tube will tend to compress the air, increasing the density. Sometimes this does not change the frequencies of the normal modes, and only affects their relative amplitudes and thus the timbre of the instrument. In other cases, the frequencies of the normal modes are affected, including the fundamental.

Chapter 203. Cylindrical Bores

Many flute instruments do, in fact, have a cylindrical bore. Many are open at the foot, with such examples as the concert flute and the tin whistle. Since the embouchure or fipple hole is also effectively an open end, these flutes act as open tubes with all harmonics, governed by Eqs. 179.1 and 179.2,

$$f_n = n\frac{s}{2L} \quad , \quad n = \text{positive integer.} \tag{203.1}$$

Some flute instruments are cylindrical and closed at the foot, such as slide whistles and some pan pipes. These are then closed tubes with primarily odd harmonics, governed by Eqs. 181.1 and 181.3,

$$f_n = m\frac{s}{2L} \quad , \quad m = \text{positive half integer} \quad , \quad n = 2m \quad . \tag{203.2}$$

Real closed-cylinder wind instruments usually have some even harmonics in their spectra, especially in the higher harmonic numbers. Deviations from being a perfect cylinder, especially the presence of tone holes, mean that our model is not perfect. But this model is still quite good, especially for understanding the fundamental.

Among reeds, the only common instrument with a cylindrical bore is the clarinet. The open foot of the clarinet does have a short, slightly flaring bell, but this has little effect on the standing wave modes. Since the reed is effectively a closed end, clarinets are therefore closed tubes governed by Eqs. 203.2. The dominance of odd harmonics in the clarinet spectrum is responsible for much of its particular timbre, often described as woody. Some smaller types of bagpipes, which have double reeds, also have a cylindrical bore.

Chapter 204. Conical Bores

Many woodwind instruments have a conical bore. A full cone shape has a cross-sectional diameter that is proportional to distance from the apex of the cone. The term conical bore actually refers to a bore having the shape of a truncated cone, that is, a cone with some amount of the apex end removed. The mathematics required to analyze the normal modes inside a cone are well beyond the level of this book.[63] We will be satisfied with just a description of the results.

As long as both ends are open, the normal mode frequencies of a conical bore are the same as for an open cylinder, described by Eqs. 203.1. The loops are all the same length, as in Table 179.1, although the amplitudes of the antinodes are not all the same. Some flute instruments fall into this category, since the embouchure or fipple hole is effectively an open end. For example, the baroque recorder is tapered to be smaller at its open foot than at the fipple.

Most reed instruments have a conical bore, this time expanding from reed to bell. Examples include the oboe, bassoon, and saxophone. Since the reed forms a closed end, the simplest appropriate model is to consider these to be full cones. Figure 204.1 shows graphs of the pressure amplitudes for standing waves in a cone, which have a surprise in store. As expected, the closed apex of the cone results in a pressure antinode. However, the constriction at the apex results in a half-loop at that end that is exactly double the normal length. The apex half-loop is the same length as the other full loops, which themselves are all the

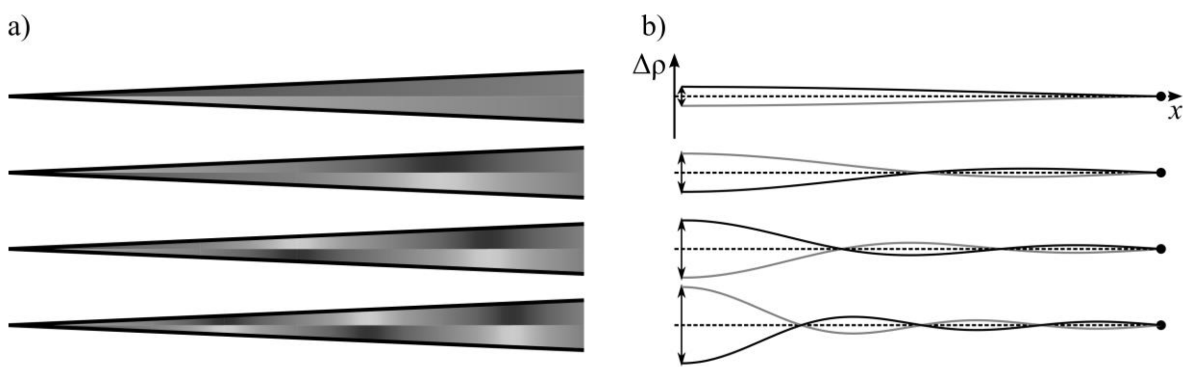

Figure 204.1

Normal mode compression standing waves in a cone. (a) Cross sections of cones contain gray scale representing density, with darker being denser. Top and bottom panels represent alternating extremes of standing wave, but gray scale does not show differing amplitudes of loops. (b) Graphs of the density extremes. Amplitudes are chosen to have same total wave energy in each standing wave.

[63] R. Dean Ayers, Lowell J. Eliason, and Daniel Mahgerefteh, "The Conical Bore in Musical Acoustics," *Am. J. Phys.* 53(6) (1985): 529–530.

same size for a given mode! As a result, Eqs. 203.1 apply, and all harmonics are possible, not just odd harmonics.

> The normal mode frequencies for a full (closed at the apex) cone are the same as for an open cylinder.

For any real conical bore reed instrument, the tip of the cone must be removed in order to add the mouthpiece. Thus, a more sophisticated model for these instruments might be a truncated cone closed at the narrow end. Here, unfortunately, our luck runs out. The normal mode frequencies for a truncated and closed cone are *not* harmonic, with modes 2 and above having frequencies higher than multiples of the fundamental.[64] This gets worse as more of the cone is truncated, and it also is worse for higher mode numbers, which is a significant concern because reed instruments tend to have many overtones in their sound spectra. One might get away with truncating as much as 5% of the cone before the inharmonicities become too bad.

In real instruments, the reed mouthpiece that replaces the cone apex has some air volume. Instrument makers go to some effort to adjust the mouthpiece so that it mimics the missing apex, as far as its influence on the standing waves. So, at the level of this book, the most effective model is to return to treating these instruments as complete cones. Of course, the instrument makers are probably not thinking about cone apexes; they are just working to get the best sound.

Chapter 205. Pipe Organ Terms

Large pipe organs are sometimes called "the king of instruments," not just because of their size, but because they incorporate the sounds of many different smaller instruments. They do this through the use of many different pipe shapes, different sound driving mechanisms, and different pipe materials. Organ pipes are organized in **ranks,** which are collections of pipes with different lengths and pitches but all the same timbre. The organist can select which ranks are active at any given time. Although not all organs include all types, there are ranks that mimic all the common types of wind instruments, and even some that are said to mimic string instruments, although they still produce sound via blowing air.

One general category of organ pipes is **flue pipes,** which are either **open** or **stopped.** Flue pipes are essentially fipple flutes, making one end effectively open. They can be cylindrical, conical, or even have rectangular cross section. An **open pipe** is also open at the non-fipple end, and therefore follows Eqs. 203.1, regardless of other aspects of its shape. **Stopped pipes** are closed at the non-fipple end; these usually have a uniform cross section, and therefore follow Eqs. 203.2.

Organ pipes are often positioned vertically, with the "mouthpiece" at the bottom. Presumably this is why that end is called the **foot** of the pipe, inconsistent with the use of that term on other instruments. To add to the confusion, a **stop** on an organ refers to the mechanism by which the player activates a rank (or group of ranks); the term has nothing to do with stopped pipes particularly.

Another common pipe category is **reed pipes.** These have some relationship to reed instruments, but the natural frequency of the reed itself also plays a significant role, similar to the reed in a harmonica. This level of complexity is outside the scope of this book.

Chapter 206. Register Hole

206a. Lower and Upper Register

When woodwind instruments can play notes based on the second or higher standing wave modes, musicians group the notes into registers. The **lower register** consists of all the notes that are based on the fundamental

[64] Ayers, "The Conical Bore in Musical Acoustics," 531.

standing wave mode. The **upper register** is the set of notes based on the second mode (which may be the second or third harmonic of the lowest mode). Some woodwinds can also access a third **extreme register.** For some instruments, the registers are given special names that apply only for that instrument, but here we will use the generic terms. (Brass instruments utilize higher modes more extensively, such that grouping pitches according to their mode is not particularly useful. Brass musicians sometime use the term **register**, but to refer to less clearly defined ranges of pitch.)

For a few flutes, such as the tin whistle, a technique called **overblowing** can sound notes in the upper register with no change in which tone holes are open or closed. Overblowing mostly just means blowing harder. If you haven't read Chapter 196, just take it as a given that this causes the instrument to prefer the second mode over the fundamental. If you have read Chapter 196, you might imagine that harder blowing forces the launch of a second pulse between times (d) and (e) in Figure 196.1. The two pulses then travel the tube separately, passing through each other, so that the total oscillation is doubled in frequency.

But for most woodwind instruments, changing register cannot be achieved solely by changing embouchure. This is not to say that an embouchure change or overblowing is not needed. But the musician needs some help from the instrument to reliably sound a note that does not include the fundamental standing wave. In other cases, the need may arise to play in the upper register softly; overblowing tends to make notes that are louder as well as higher pitched.

In order to eliminate the fundamental mode but allow others, for instance the second mode, the key is to force a node to exist where the allowed mode would have one. Since the fundamental mode has no nodes (except at the ends), it cannot exist while this new node is enforced. Figure 206.1 shows the first five standing wave modes that are normally possible in open tube (which might represent a flute, for instance). If we force a node to occur in the center, then the modes labeled (b), (d), and (f) cannot occur. (Chapter 210 describes a very similar technique for strings.)

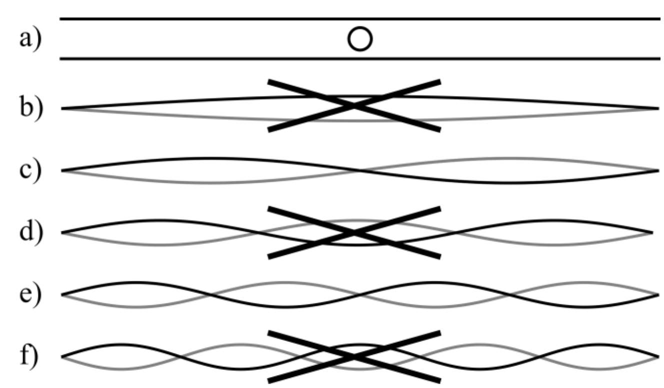

Figure 206.1
Standing wave modes that are disallowed by opening of a register hole.

Modes other than the fundamental can also become disallowed. In Figure 206.1, only the even harmonic modes remain. Since these themselves form a harmonic series, one might say that the second harmonic mode of the tube becomes the fundamental partial of the harmonic sound produced.

The way that woodwinds create nodes is by opening a hole on the side of the tube. This **register hole** appears identical to a tone hole, but it functions in a slightly different way. In the simplest model, opening a register hole connects that location inside the tube to the outside air, fixing its pressure to create the node. The difference from a tone hole is that a standing wave can exists on both sides of the hole. Since the pressure node is a displacement antinode, the two sides are not isolated from each other.

The same pitch could be reached by opening tone holes all along the lower half of the tube. One difference between that and opening a register hole is a subtle difference in timbre, which this book will not explore further. But more significantly, because the register hole allows the standing wave to fill the entire tube, the standing wave can be further affected by the tone holes in the lower portion of the tube.

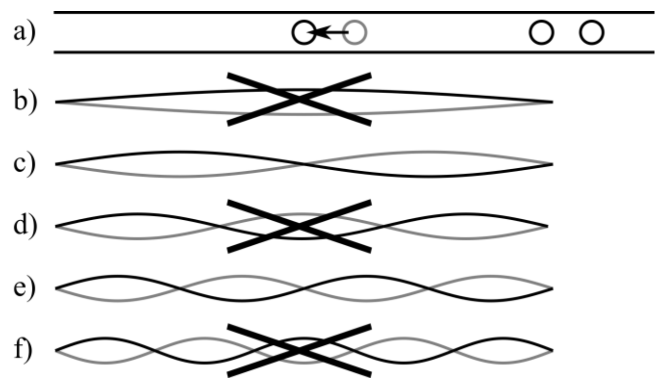

Figure 206.2

Changing to upper register when some tone holes are also open. Ideal location for register hole moves.

Figure 206.2 shows the effect of the register hole when a few tone holes are also open, thus removing from action a short piece at the foot of the instrument. A problem is immediately apparent: for this new pitch, the node of the second mode has moved, so that the ideal location for the register hole has moved as well. However, it is not easy to engineer a hole that can slide its position along a solid tube!

It turns out that an effective solution to this is to make the register hole rather small. In this way, the register hole "mostly" fixes the pressure at its location, but allows the forced node to be somewhat displaced from the actual register hole location. The register hole on a real instrument is located roughly 5/8 of the tube length from the foot, midway between the ideal positions for the full tube and half of the tube. On many woodwinds, the register hole for the upper register can be identified as the one operated by the thumb of the upper hand, and therefore placed on the opposite side of the tube from the tone holes. But these are just choices of instrument construction, not required by the physics of the operation.

Most woodwinds have normal modes at all harmonics of the fundamental. As a result, the upper register is based on the second harmonic. Upper register frequencies are double those of the lower register, musically one octave higher. But for the clarinet, the only common concert instrument with only odd harmonic normal modes, the upper register is based on the third harmonic mode. Notes in the lower and upper registers with otherwise equivalent tone holes covered therefore differ by an octave plus a fifth, i.e., a frequency factor of three. The register hole on a clarinet is also positioned further from the bell than on other woodwinds, corresponding to the location of the node of the third harmonic mode.

The saxophone has a special feature: it is the only common instrument for which an attempt has been made for the register hole to be moveable, sort of, as suggested by Figure 206.2. There are two register holes for the upper register. They are both activated by the same key on the instrument, but only one or the other opens at a time, depending on which lower tone holes are open or closed. This choice is made by a mechanism in the keys, so that the musician can think of the register key as having a single function.

On less mechanically complex instruments, such as the recorder, there is sometimes not a clear distinction between the tone holes and the register hole. Notes in the upper registers may be achieved by using **forked fingerings,** in which a tone hole or two are left open in between others that are closed. These upper tone holes are functioning exactly as register holes, forcing a node or two in the middle of the tube.

206b. Extra: Extreme Register

For woodwinds with all harmonic normal modes available, one could in principle use the third mode to raise the notes from the lower register up one octave and a fifth. However, the usual method is to play all the notes between one octave and two octaves above the lowest note by using the second mode, that is, in the upper register. From the musician's perspective, the upper register is therefore like a duplicate of the lower register, just one octave higher.

Clarinets have a specific extreme register hole for notes in the extreme register, which is based on the third standing wave mode (the fifth harmonic mode, up two octaves and a major third). On other instruments, playing in the extreme register is likely to be achieved using forked fingerings. In cases where the upper register is used all the way to two octaves above the lowest note, the extreme register is based on the fourth

Table 207.1

Major categories of wind instruments, based on the energy source. Center columns list the methods *typically* used to obtain different pitches. Examples of each case are on the right.
In the Mode Adj. column, E = embouchure adjustment, R = register holes, and O = overblowing.

Category	Subcategories		Mode Adj.	Length Adj.	Examples
brass			E	—	natural horn, bugle
			E	slide	trombone
			E	valves	trumpet, horn, tuba
woodwinds	reeds	single reed	R	tone holes	clarinet, saxophone
		double reed	R	tone holes	oboe, bassoon, bagpipes
	flutes	fipple flute	—	slide	slide whistle
			—	many pipes	flue pipes of pipe organ
			O	tone holes	tin whistle
			R	tone holes	recorder
		direct blown	O	many pipes	pan pipes
			O(R)	tone holes	flute

mode, skipping the third mode entirely. But for these high notes, different instruments use a wide variety of methods, so that it is hard to find generalities.

Chapter 207. Wind Instrument Summary

Table 207.1 shows the breakdown of categories graphically. It also shows methods of obtaining different pitches that are commonly associated with each of the categories. With one exception, these are *only* commonalities, not restrictions; there are less-common instruments that make other choices.

The exception is that only brass instruments are able to control the mode solely through the **embouchure,** the way that the musician holds and uses his or her mouth. With woodwinds, a change in embouchure may be necessary when playing modes above the fundamental, but the embouchure change is not the primary cause of the pitch change. Instead, woodwinds can sometimes play using modes a few above the fundamental, with the methods **overblowing** and **register holes** described in Chapter 206.

The business of classifying musical instruments according to their characteristics has a very long history. Today, the **Hornbostel-Sachs** system[65] is the most used. It goes into much more detail than Table 207.1. It also encompasses all possible musical instruments, not just wind instruments.

Chapter 208. String Instrument Vibration Sources

Although the length and other properties of a string define which frequencies it can vibrate at, they will not actually vibrate until energy is provided by a musician. The energy provided is never oscillatory, because a human cannot directly vibrate something at a frequency high enough to be heard. There are two general categories of ways to provide the energy.

1. The string can be deflected from its equilibrium state, and then released. Since the vibration of the string occurs after the release, the musician's control of the timbre is limited to, at most, any consequences of the shape of the initial deflection. The deflection is applied at one point along the string, so that the shape of the string as it is released is certainly not sinusoidal, but instead quite close to a triangle. The subsequent vibration must therefore be the superposition of several normal modes.

[65] Erich Moritz von Hornbostel and Curt Sachs, "Systematik der Musikinstrumente. Ein Versuch," *Zeitschrift für Ethnologie* 46 (1914): 553–590.

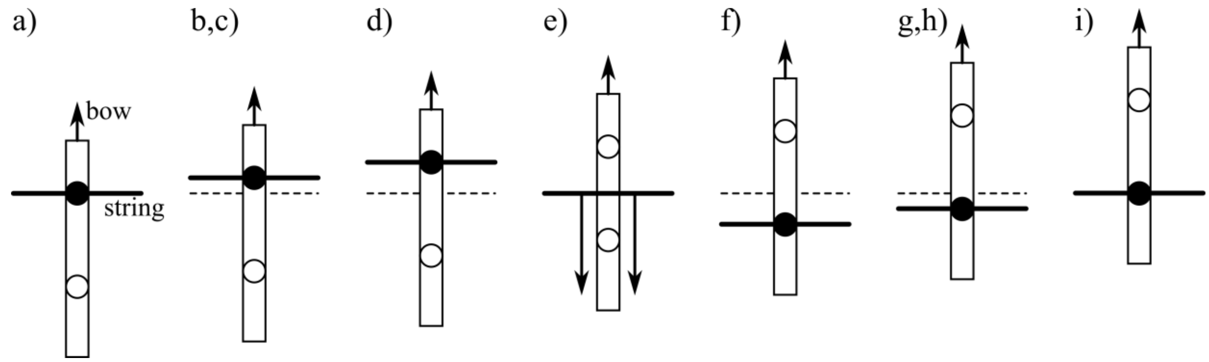

Figure 208.1

Stick-slip motion of a string as it is bowed. Parts are labeled to match parts of Figure 209.1. Filled dots indicate string stuck on the bow. String is moving with bow, except in snapshot (e) when the string is moving downwards.

This category can be divided into two subcategories, based on whether the string is **plucked** or **struck**. In plucked instruments, such as the guitar and harpsichord, there is slightly more delay between deformation and release. Struck string instruments, such as the hammered dulcimer and piano, are sometimes classified as percussion instruments. Although percussion is certainly involved, and this classification may be useful in some musical contexts, for the purposes of understanding their sound it is more helpful to include them as string instruments.

2. The string can be deflected sideways by rubbing with another object. This is called the **bowed** category, because the other object is most commonly a bow. A **bow** is a rigid stick with a flexible material (traditionally horse hair) stretched from one end to the other so that it is not touching the stick. The flexible material is then rubbed across the string of the instrument. There are a few bowed instruments, however, which do not use an actual bow. On a hurdy-gurdy, for example, the strings are "bowed" by a wheel that rubs against the strings.

   The bow is rubbed across the string in a steady motion. In the simplest situation, this would simply deflect the string. However, the bow is made slightly sticky, usually with **rosin,** which results in the **slip-stick motion** illustrated in Figure 208.1. Initially (parts a–d) the string moves stuck to the bow. When the string reaches a large enough deflection (d), the restoring force releases it from the bow, and it proceeds to slip along the bow as it moves toward the equilibrium position (e). Once the string slows down, it sticks to the bow again (f). It then (f–i) travels along with the bow until it returns to the equilibrium position, and the cycle restarts. In this way, a vibratory string motion is generated from the steady motion of the bow.

Chapter 209. Bow Feedback

For the bowing action illustrated in Figure 208.1, if we knew how sticky the bow was, and if we assumed how the restoring force on the string increased with displacement, then maybe we could determine how far the string would be deformed before unsticking from the bow. Combine this with the speed of the bow, and perhaps we could find the period of one cycle.

But there is something missing. The whole point of a string instrument is that the frequency should be determined by the length and properties of the string, not by the method of driving the vibration. There must be some feedback from the string, to tell the bow when to slip and when to stick. (This is also a good thing for the musician, because controlling the bow speed as well as the sticking, which depends on the force of the bow on the string, with the necessary precision would be nearly impossible!)

Helmholtz discovered[66] that when bowed with the proper amount of force on the string, the motion of a bowed string is as depicted in Figure 209.1. At any moment, the string has a triangular shape; the dashed curves trace the path of the tip of the triangle. In agreement with Figure 208.1, the string is stuck to the bow at all times except the interval (d) to (f). While the string sticks and slips on the bow, the triangular shape travels back and forth between the ends. Notice that the pulse inverts at each reflection, as appropriate for a fixed end. The time for one cycle is equal to the period of the fundamental mode of the string.

$$T = \frac{2L}{s} \quad . \tag{209.1}$$

The feedback arises when the triangular pulse reaches the bow. At that time, the string jerks downwards, so that the transition from sticking to slipping is determined by the shape of the standing wave, instead of by the stickiness of the bow. As long as the player applies roughly the correct force on the bow, this feedback will ensure the appropriate frequency.

A comparison of Figure 209.1 and the air reed feedback in Figure 196.1 is worth noting. The driving mechanism is very different, but both involve pulses traveling along the medium and inverting at the ends. The two figures do not start at the same point in the cycle; can you figure out how they match up?

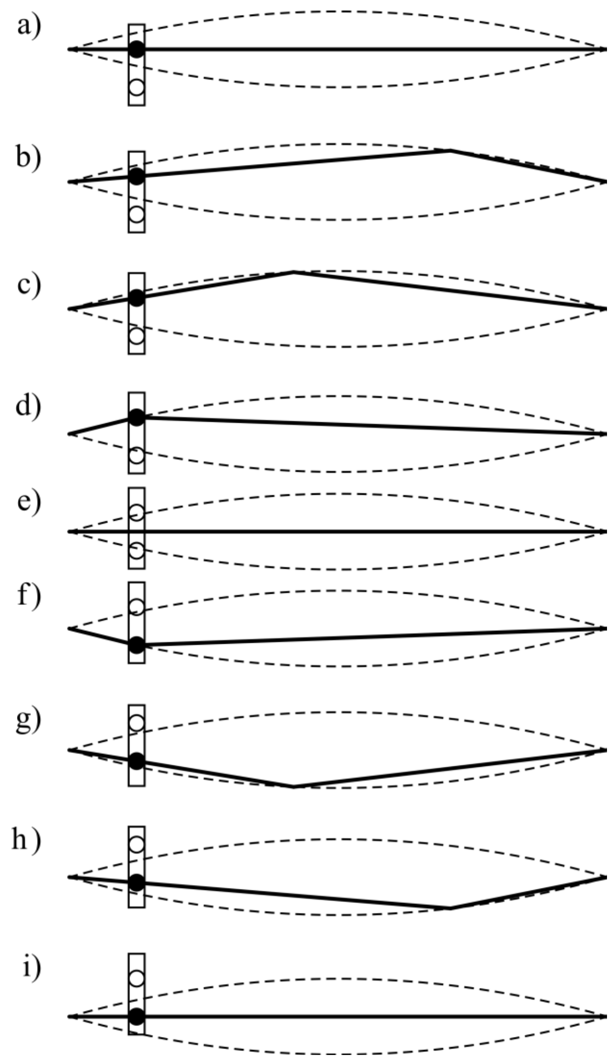

Figure 209.1

Motion of bowed string during vibration cycle. Parts are labeled to match parts of Figure 208.1. (d–f) Time between these snapshots, while string is slipping along bow, is much smaller than between others.

Chapter 210. Playing String Harmonics

Once a plucked or struck string is vibrating, it of course can be stopped by placing an object on the string, especially something soft that will rapidly absorb the vibration energy. But in order for that energy transfer to take place, the string has to be moving at the location where it is touched. If a string is vibrating in a mode other than the fundamental, the string can be touched at a node without damping the vibration.

When a string is vibrating with a superposition of modes, each one vibrates on its own. This leads to a rule that is sort of the inverse of the one in Chapter 211.

> When a vibrating string is touched at a point, so that point is forced to stop moving, modes that have a node at that point can continue to vibrate.

[66] Hermann von Helmholtz, *On the Sensations of Tone as a Physiological Basis for the Theory of Music,* trans. Alexander J. Ellis (London: Longmans, Green, and Co., 1885), 384–387.

Guitar players know this as the technique of **playing harmonics.** Either when a string is plucked, or afterwards, a finger is held on the string very lightly. A light touch is required in order to limit the touch to as short a length of the string as possible. The fundamental, and possibly other modes, will be stopped from sounding. But if the touch is positioned at a node for a higher harmonic, then that pitch will continue.

The same pitch could have been obtained by simply holding the string fixed at that point. But playing harmonics has a distinctive, airy timbre. If you have read Chapter 206, then you will notice the very close similarity to the technique of using register holes on woodwind instruments. There, too, the same pitch could be achieved by simply shortening the active part of the tube, but the results are not the same in timbre.

Chapter 211. Avoiding String Harmonics

When a string is plucked or struck, the initial shape is essentially a triangle between the two end points and the point where the string is plucked or struck. The theory of Fourier analysis assures us that this shape can be obtained as a superposition of sinusoidal shapes. Each of those contributing shapes is then the initial displacement for a normal mode that will start vibrating when the string is released. Although this book does not cover how to determine which sinusoids are needed to make the triangle, we will here invoke a rule that will probably not seem surprising.

> When a string is deformed into a triangle, the Fourier analysis does not include any sinusoid which has a node at the location of maximum deflection.

This is sort of the inverse of the rule in Chapter 210. As a result, a string plucked in the middle will produce a sound that has no contribution from the even harmonics. A string plucked one-sixth of the way from one end will produce a sound that has no contribution from the modes with a mode number that is a multiple of six.

All of these options will include the fundamental, because the fundamental mode has nodes only at the string ends, and thus is never excluded by the rule. So regardless of where the string is plucked, the frequency of the total motion will be the same. But the timbre of the sound is subtly affected. This is a technique used by classical guitarists, in particular.

Chapter 212. Soundboards

Strings make nice standing waves, easy to model, easy to see, and somewhat easier to control than tubes of air. However, strings are not very good at creating sound waves. As a string vibrates, it tends to cut through the air rather than push it. An electric guitar illustrates this very nicely. If it is not plugged in to an amplifier, then plucking a string makes very little sound.

Electric guitars solve this problem by not even attempting to use the energy in the vibrating string to create sound. One or more **magnetic pickups** for each string detect vibrations as the metallic string disturbs a magnetic field, but doing so removes a negligible amount of energy from the string. The detected vibration is then electrically amplified to drive a speaker. If you have read Chapter 106, then you have seen a bit about the connection between magnets and electrical circuits. But magnetic pickups rely on an additional physical effect that is not described in this book.

For a non-electric string instrument to make a reasonable volume, there must be a way for the vibrational energy to get out of the string. String instruments accomplish this through a **soundboard.** In this book, this is used as the generic term for a flat surface that vibrates in response to the string vibration. It is not always wooden, and in some cases (such as the head of a banjo) it is not even rigid. But vibrating a large, flat area is a good way to make sound.

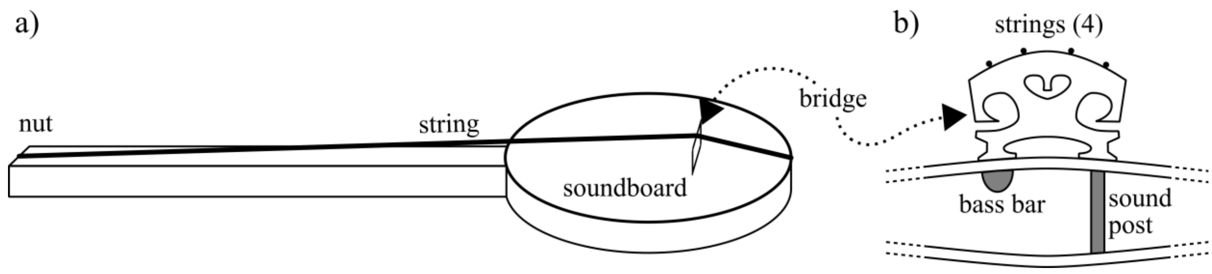

Figure 212.1

(a) Schematic of a generic string instrument (looking somewhat like a banjo). (b) Cross section of violin body, including the bridge (not to scale). Strings are seen in cross section.

Since string waves are transverse, it is easiest to transfer the oscillation if the strings lie parallel to the surface of the soundboard. A **bridge** is placed between the strings and soundboard, as shown in Figure 212.1(a). The bridge has several functions. In many instruments, the bridge serves to hold the strings away from the soundboard, so that they do not hit it as they vibrate. The bridge defines one end of the string length, the other end being at the **nut.** But the most important function is to transfer vibrations from the string into the soundboard.

For plucked or struck strings, the bridge (and the sound board underneath it) is designed as fairly rigid, with the result that the energy transfer out of the string is relatively slow. This allows the string to continue to vibrate after being energized, thus sustaining the note. Bowed instruments can sustain a note through continuous playing, so the bridge is designed to be relatively flexible, allowing it to rapidly transfer energy to the sound board. The string therefore stops vibrating rapidly when the driving force is removed. Figure 212.1(b) shows a violin bridge as an example. The complex shape is not just for decoration, but provides the required flexibility. But whether the energy transfer is fast or slow on the human timescale, it is important that it remain slow compared to one vibration period. In the language of Chapter 99 the string must be underdamped by the bridge. This is important so that the string retains clear resonances, which control vibration frequency.

In order to transfer the vibrations, the bridge must move slightly, so that the end of the string is not quite a perfect node. However, the string being underdamped implies that the end is very close to fixed. This also ensures that there is no significant feedback from the soundboard to the strings. This is of course important, because the strings are supposed to determine the pitches played. It is therefore appropriate to model the bridge and soundboard as a transducer with some response curve for converting string energy into sound power.

The soundboard itself is a medium for two-dimensional standing waves, and will have normal modes of vibration as described in Chapter 187. It may be as simple as the head of a banjo, a uniform membrane with a nodal line forced by attachment around the rim. Or it may be as complex as the shape of a violin, including a thickness that varies from the centerline to the edges. The guitar complicates things with **bracing,** strips of wood attached to the interior surface of the soundboard, which stiffen it in particular directions. The violin also has one such strip called the **bass bar,** the cross section of which appears in Figure 212.1(b). All these modifications make it extremely difficult to predict the shape and frequency of the soundboard's normal modes, although they still must follow the rules from Chapter 187.

Generally speaking, the resonances in the soundboard response curve should not be too pronounced. The vibrational spectrum of the string is the input that determines the pitch, while the response curve of the soundboard molds the output spectrum to have a specific timbre. Two factors help ensure that the soundboard responds at all the pitches that can be played. One is that the resonances are broadened and shortened (that is, given a moderately low Q, certainly less than 100) by damping, preferably in the form of sound energy leaving the soundboard. Keeping Q low also prevents too long ringing of the instrument body when the string stops vibrating. The other factor influencing the response curve is that the added freedom of two dimensions leads to a greater number of normal modes than the harmonic series typical of a 1D medium. The greater number of resonance peaks means that they overlap more at non-resonant frequencies.

Some instruments, such as the guitar and violin, have two soundboards, forming the back and front of the **body** of the instrument. This can be as simple as it looks from the outside. But sometimes, especially for bowed instruments, hidden inside the body is a **sound post** connecting the two soundboards. In the violin family the sound post is placed directly below one leg of the bridge (see Figure 212.1(b)), so that the bridge directly drives both soundboards. In other cases, the connection is less extreme.

Chapter 213. String Instrument Bodies

Many stringed instruments have an enclosed volume under the soundboard, which is the **body** of the instrument. When there are two soundboards this volume is between them, but the body can take many shapes. The purpose of the body is to provide a resonance in the enclosed air, thus assisting the soundboard in turning the string vibrations into sound. There is always, therefore, at least one hole into the body, allowing it to act as a Helmholtz resonator. This hole (or holes) is called the **sound hole.** Many instruments, most notably members of the violin family, have a sound hole with a very particular and unusual shape called an **f-hole**, named for its resemblance to a lower case, italic *f*.

The air in the body can oscillate in other modes as well, and those modes are surprisingly similar to the normal modes of a tube, considering how un-tube-like the body is. Figure 213.1 shows the five lowest-frequency modes, other than the Helmholtz resonator mode, for the air in a guitar with the soundboards immobilized.[67] Different models of guitar have different body shapes, and therefore the mode frequencies can vary by up to 10% even for guitars of similar size. The figure shows typical frequency values. The shading does not show the pressure amplitude of the oscillations. As with figures for two-dimensional standing waves (see Chapter 187), it only shows which parts are in phase with each other, with the boundaries being nodal lines. When the gray areas are compressed, the white are rarefied, and vice versa. All of these modes have “ends” which are the closed walls of the guitar body, and which therefore form pressure antinodes.

There are a few interesting features to notice.

- Modes (a) and (b) are very similar to the fundamental mode of a doubly closed tube, vertical and horizontal respectively. Using Eq. 179.1 with the typical length and width of a guitar body does a decent job of predicting their frequencies, although the prediction is about 15% lower than the actual frequencies.

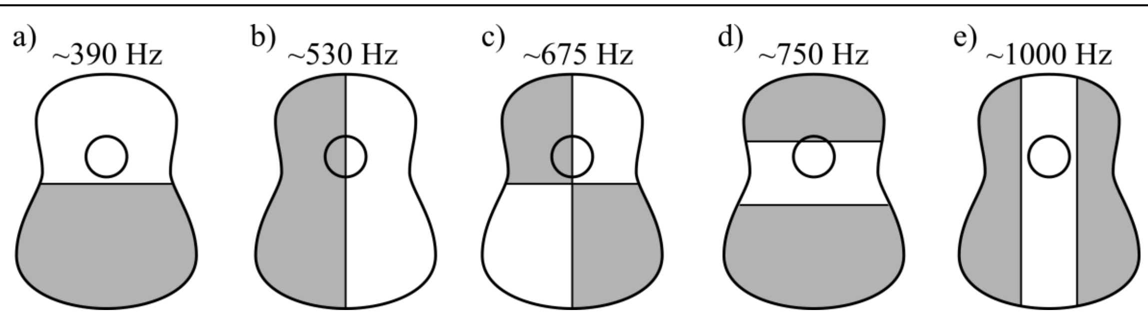

Figure 213.1

Lowest non-Helmholtz normal modes for air in guitar body, from lowest to higher frequency. Gray and white are out of phase, boundaries are nodal lines of air pressure. Frequencies are typical for full sized guitar bodies.

[67] Thomas D. Rossing, John Popp, and David Polstein, “Acoustical Response of Guitars,” *SMAC 83* (1985): 323–324.

- Modes (d) and (e) are very similar to the second harmonic modes of a vertical and horizontal doubly closed tube. The loops (that is, the distance between nodes) are not of equal length, which is an effect of the sound hole. The frequencies are a little less than twice the frequencies of modes (a) and (b), but still a little above the frequencies predicted using Eqs. 179.1 and 179.2.
- The Helmholtz resonator mode, sometimes called the **breathing mode** in this context, has a typical frequency near 120 Hz. This has no significant relationship to the modes in Figure 213.1, except that the breathing mode has a lower frequency. Comparing Eq. 28.1 and Eq. 179.1, they depend on quite different characteristics of the guitar body.

The normal mode in Figure 213.1(c) reminds us that a simple tube model cannot capture the full complexity of the situation. The picture of mode (c) appears like a superposition of the pictures of modes (a) and (b). However, that cannot be the true origin of mode (c), because all three have different frequencies. If (a) and (b) were superposed, they would rapidly go in and out of phase with each other, quite unlike the evolution of (c).

Although these air standing waves are important conceptually, they cannot be used in isolation. There is one more level of complexity that must be addressed to obtain a useful model for the resonating body of a string instrument. The problem is that the pressure of the air standing waves places oscillating forces on the soundboard(s), and in return the vibration modes of the soundboard(s) apply forces on the enclosed air. This is why it was important that the modes in Figure 213.1 were obtained with the soundboards immobilized.

The situation is somewhat like feedback. The drive, or source of oscillation energy, is the (top) soundboard. This causes the air to oscillate, but that oscillation feeds back to affect the soundboard. If there is a bottom soundboard, it is driven by the top soundboard (if there is a sound post) or by the air, but then also pushes back on the air.

A crucial difference from the feedback described in Chapter 195 and following is that in those cases the energy source had no preferred frequency (or in the case of lips on a brass instrument, weakly preferred). Here, both the drive and driven objects have multiple definite resonant frequencies. Modeling such situations requires the theory of **coupled oscillators,** which is beyond the scope of this book. This theory is required when two (or more) oscillating systems interact through a force that is weaker than their internal restoring forces, but not so weak that it can be neglected. Generally, the result is that the whole combination has a new, different set of natural frequencies and normal modes. It is possible to relate those new frequencies to the old natural frequencies of the component systems, but not without quite a bit of difficult math.

We'll therefore settle for the following level of understanding. The soundboard(s) and enclosed air of the body can each support standing waves, in ways that are sometimes recognizable from simpler systems. When they combine, they work together to find new modes of cooperative motion, each with a specific natural frequency. Practically speaking, it might be easier to experimentally measure those new frequencies, than to calculate them from the natural frequencies of the individual parts. But regardless of that, the combined body of the instrument can be driven by the string through the bridge, and as a result translate the vibration of the string into sound in the air.