points. Both these theorems, of which the latter can be regarded as a generalization of the former, will be demonstrated both analytically and by simple geometrical considerations. *The excess of the sum of the angles of a triangle formed by shortest lines over two right angles is equal to the total curvature of the triangle.* [...]

Evidently we can express this important theorem thus also: the excess over the two right angles of the angles of a triangle formed by shortest lines is to eight right angles as the part of the surface of the auxiliary sphere, which corresponds to its as its integral curvature, is to the whole surface of the sphere. In general, the excess over $2n - 4$ right angles of the angles of a polygon of n sides, if these are shortest lines, will be equal to the integral curvature of the polygon.

15.B Gauss's Correspondence

15.B1 Three letters between Gauss and Sophie Germain

(*a*) *Germain to Gauss, 21 November 1804*

Germain corresponded with Gauss on questions concerning number theory using the pseudonym Antoine Le Blanc.

Your *Disquisitiones Arithmeticae* have been the object of my admiration and my study for a long time. The last chapter of this book contains, amongst other remarkable things, the beautiful theorem about the equation $4\frac{(x^n - 1)}{x - 1} = y^2 \pm nz^2$; I believe that it can be generalized. [She went on to describe how, and continued as follows.]

I add to this art some other considerations which relate to the famous equation of Fermat $x^n + y^n = z^n$ whose impossibility in integers has still only been proved for $n = 3$ and $n = 4$; I think I have been able to prove it for $n = p - 1$, p being a prime number of the form $8k + 7$. I shall take the liberty of submitting this attempt to your judgement, persuaded that you will not disdain to help with your advice an enthusiastic amateur in the science which you have cultivated with such brilliant success....

(*b*) *Germain to Gauss, 20 February 1807*

Germain used her influence to help ensure that Gauss came to no harm when the Napoleonic War brought danger to Brunswick, but her emissary inadvertently revealed her true name. She then wrote to explain.

The consideration due to superior men will explain the care I have taken to ask General Pernety to make it known to whomever he thought appropriate that you have the right to the esteem of any enlightened government.

In describing the honourable mission I charged him with, M. Pernety informed me that he had made known to you my name. This has led me to confess that I am not as completely unknown to you as you might believe, but that fearing the ridicule attached to a female scientist, I have previously taken the name of M. LeBlanc in communicating to you those notes that, no doubt, do not deserve the indulgence with which you have responded.

The appreciation I owe you for the encouragement you have given me, in showing me that you count me among the lovers of sublime arithmetic whose mysteries you have developed, was my particular motivation for finding out news of you at a time when the troubles of the war caused me to fear for your safety; and I have learned with complete satisfaction that you have remained in your house as undisturbed as circumstance would permit. I hope, however, that these events will not keep you too long from your astronomical and especially your arithmetical researches, because this part of science has a particular attraction for me, and I always admire with new pleasure the linkages between the truths exposed in your book. Unfortunately, the ability to think with force is an attribute reserved for a few privileged minds, and I am sure that I will not encounter any of the developments that you deduce, seemingly so effortlessly, from those that you have already made known.

I include with my letter a note intended to show you that I have maintained an appetite for analysis that the reading of your work has inspired, and that has continually provided me with the confidence to send you my feeble attempts, without any other recommendation to you than the goodwill accorded by scientists to admirers of their work.

I hope that the information that I have today confided to you will not deprive me of the honour you have accorded me under a borrowed name, and that you will devote a few minutes to write me news of yourself.

(*c*) *Gauss to Germain, 30 April 1807*

Your letter of February 20, which did not arrive until March 12, was for me the source of as much pleasure as surprise. How pleasant and heartwarming to acquire a friend so flattering and precious. The lively interest that you have taken in me during this war deserves the most sincere appreciation. Your letter to General Pernety would have been most useful to me, if I had needed special protection on the part of the French government.

Happily, the events and consequences of war have not affected me so much up until now, although I am convinced that they will have a large influence on the future course of my life. But how can I describe my astonishment and admiration on seeing my esteemed correspondent M. LeBlanc metamorphosed into this celebrated person, yielding a copy so brilliant it is hard to believe? The taste for the abstract sciences in general and, above all, for the mysteries of numbers, is very rare: this is not surprising, since the charms of this sublime science in all their beauty reveal themselves only to those who have the courage to fathom them. But when a woman, because of her sex, our customs and prejudices, encounters infinitely more obstacles than men in familiarizing herself with their knotty problems, yet overcomes these fetters and penetrates that which is most hidden, she doubtless has the most noble courage, extraordinary talent, and superior genius. Nothing could prove to me in a more flattering and less equivocal way that the attractions of that science, which have added so much joy to my life, are not chimerical, than the favour with which you have honoured it.

The scientific notes with which your letters are so richly filled have given me a thousand pleasures. I have studied them with attention, and I admire the ease with which you penetrate all branches of arithmetic, and the wisdom with which you generalize and perfect. I ask you to take it as a proof of my attention if I dare add a remark to your last letter. It seems to me that the inverse proposition 'If the sum of the nth powers of two numbers is of the form $hh + nff$, then the sum of the numbers themselves will be of the same form' is put a little too strongly. Here is an example of where this rule fails:

$$\begin{aligned} 15^{11} + 8^{11} &= 8649755859375 + 8589934592 \\ &= 8658345793967 \\ &= (1595826)^2 + 11(745391)^2 \end{aligned}$$

Nevertheless $15 + 8 = 23$ cannot be reduced to the form $xx + 11yy$.