15.A Gauss's Mathematical Writings

15.A1 Gauss's mathematical diary for 1796

[1] The principles upon which the division of the circle depend, and geometrical divisibility of the same into seventeen parts, etc. March 30 Brunswick.

[2] Furnished with a proof that in case of prime numbers not all numbers below them can be quadratic residues. April 8 Brunswick.

[3] The formulae for the cosines of submultiples of angles of a circumference will admit no more general expression except into two periods. April 12 Brunswick.

[4] An extension of the rules for residues to residues and magnitudes which are not prime. April 29 Göttingen.

[5] Numbers which can be divided variously into two primes. May 14 Göttingen.

[6] The coefficients of equations are given easily as sums of powers of the roots. May 23 Göttingen.

[7] The transformation of the series $1 - 2 + 8 - 64 + \ldots$ into the continued fraction

$$\cfrac{1}{1+\cfrac{2}{1+\cfrac{2}{1+\cfrac{8}{1+\cfrac{12}{1+\cfrac{32}{1+\cfrac{56}{1+128\ldots}}}}}}}$$

$$1 - 1 + 1\cdot3 - 1\cdot3\cdot7 + 1\cdot3\cdot7\cdot15 + \ldots = \cfrac{1}{1+\cfrac{1}{1+\cfrac{2}{1+\cfrac{6}{1+\cfrac{12}{1+28\ldots}}}}}$$

and others. May 24 Göttingen.

[8] The simple scale in series which are recurrent in various ways is a similar function of the second order of the composite of the scales. May 26.

[9] A comparison of the infinities contained in prime and compound numbers. May 31 Göttingen.

[10] A scale where the terms of the series are products or even arbitrary functions of the terms of arbitrarily many series. June 3 Göttingen.

[11] A formula for the sum of factors of an arbitrary compound number: general term

$$\frac{a^{n+1} - 1}{a - 1}.$$

June 5 Göttingen.

[12] The sum of the periods when all numbers less than a [certain] modulus are taken as elements: general term $[(n+1)a - na]a^{n-1}$. June 5 Göttingen.

[13] Laws of distributions. June 19 Göttingen.

[14] The sum to infinity of factors $= \frac{\pi^2}{6} \cdot$ sum of the numbers. June 20 Göttingen.

[15] I have begun to think of the multiplicative combination (of the forms of divisors of quadratic forms). June 22 Göttingen.

[16] A new proof of the golden theorem all at once, from scratch, different, and not a little elegant. June 27

[17] Any partition of a number a into three $\square$ gives a form separable into three $\square$. July 3

[17a] The sum of three squares in continued proportion can never be a prime: a clear new example which seems to agree with this. Be bold! July 9

[18] EUREKA. number $= \triangle + \triangle + \triangle$. July 10 Göttingen.

[19] Euler's determination of the forms in which composite numbers are contained more than once. [July Göttingen]

[20] The principles for compounding scales of series recurrent in various ways. July 16 Göttingen.

[21] Euler's method for demonstrating the relation between rectangles under line segments which cut each other in conic sections applied to all curves. July 31 Göttingen.

[22] $a^{2n+1(p)} \equiv 1$ can always be solved. August 3 Göttingen.

[23] I have seen exactly how the rationale for the golden theorem ought to be examined more thoroughly and preparing for this I am ready to extend my endeavours beyond the quadratic equations. The discovery of formulae which are always divisible by primes: $\sqrt[n]{1}$ (numerical). August 13 Göttingen

[24] On the way developed $(a + b\sqrt{-1})^{m+n\sqrt{-1}}$. August 14.

[25] Right now at the intellectual summit of the matter. It remains to furnish the details. August 16 Göttingen.

[26] $(a^p) \equiv (a) \bmod p$, a the root of an equation which is irrational in any way whatever. [August] 18.

[27] If P, Q are algebraic functions of an indeterminate quantity which are incommensurable. One is given $tP + uQ = 1$ in algebra as in number theory. [August] 19 Göttingen.

[28] The sums of powers of the roots of a given equation are expressed by a very simple law in terms of the coefficients of the equation (with other geometric matters in the Exercitiones). [August] 21 Göttingen.

[29] The summation of the infinite series

$$1 + \frac{x^n}{1 \ldots n} + \frac{x^{2n}}{1 \ldots 2n} \text{ etc.}$$

same day August 21.

[30] Certain small points aside, I have happily attained the goal namely if $p^n \equiv 1 \pmod{\pi}$ then $x^\pi - 1$ is composed of factors not exceeding degree n and therefore a sum of conditionally solvable equations; from this I have deduced two proofs of the golden theorem. September 2 Göttingen.

[31] The number of different fractions whose denominators do not exceed a certain bound compared to the number of all fractions whose numerators or

denominators are different and less than the same bound when taken to infinity is $6:\pi^2$. September 6.

[32] If $\int^{[x]} \frac{dt}{\sqrt{(1-t^3)}}$ is denoted $\Pi(x) = z$, and $x = \Phi(z)$ then

$$\Phi(z) = z - \frac{1}{8}z^4 - \frac{1}{112}z^7 - \frac{1}{1792}z^{10} + \frac{3z^{13}}{1792 \cdot 52} - \frac{3 \cdot 185 z^{16}}{1792 \cdot 52 \cdot 14 \cdot 15 \cdot 16} \cdots$$

September 9.

[33] If $\Phi\left(\int \frac{dt}{\sqrt{(1-t^n)}}\right) = x$ then

$$\Phi{:}z = z - \frac{1 \cdot z^n}{2 \cdot n + 1}A + \frac{n - 1 \cdot z^n}{4 \cdot 2n + 1}B - \frac{nn - n - 1[z^n]}{2 \cdot n + 1 \cdot 3n + 1}C \ldots.$$

[34] An easy method for obtaining an equation in y from an equation in x, if given

$$x^n + ax^{n-1} + bx^{n-2} \ldots = y.$$

September 14.

[35] To convert fractions whose denominator contains irrational quantities (of any kind?) into others freed of this inconvenience. September 16.

[36] The coefficients of the auxiliary equation for the elimination are determined from the roots of the given equation. Same day.

[37] A new method by means of which it will be possible to investigate, and perhaps try to invent, the universal solution of equations. Namely by transforming into another whose roots are $\alpha\rho' + \beta\rho'' + \gamma\rho''' + \ldots$ where $\sqrt[n]{1} = \alpha, \beta, \gamma$ etc. and the number n denotes the degree of the equation. September 17.

[38] It seems to me the roots of an equation $x^n - 1$ [$=0$] can be obtained from equations having common roots, so that principally one ought to solve such equations as enjoy rational coefficients. September 29 Brunswick.

[39] The equation of the third degree is this:

$$x^3 + xx - nx + \frac{n^2 - 3n - 1 - mp}{3} = 0$$

where $3n + 1 = p$ and m is the number of cubic residues omitting similarities. From this it follows that if $n = 3k$ then $m + 1 = 3l$, if $n = 3k \pm 1$, then $m = 3l$. Or

$$z^3 - 3pz + pp - 8p - 9pm = 0.$$

By these means m is completely determined, $m + 1$ is always $\square + 3\square$. October 1 Brunswick.

[40] It is not possible to produce zero as a sum of integer multiples of the roots of the equation $x^p - 1 = 0$. ⊙ October 9 Brunswick.

[41] Obtained certain things concerning the multipliers of equations for the elimination of certain terms, which promise brightly. ⊙ October 16 Brunswick.

[42] Detected a law: and when it is proved a system will have been led to perfection. October 18 Brunswick.

[43] Conquered GEGAN. October 21 Brunswick.

[44] An elegant interpolation formula. November 25 Göttingen.

[45] I have begun to convert the expression

$$1 - \frac{1}{2^\omega} + \frac{1}{3^\omega}$$

into a power series in which ω increases. November 26 Göttingen.

[46] Trigonometric formulae expressed in series. By December.

[47] Most general differentiations. December 23.

[48] A parabolic curve is capable of quadrature, given arbitrarily many points on it. December 26.

[49] I have discovered a true proof of a theorem of Lagrange. December 27.