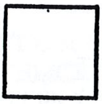
THE GENERATION OF CORRESPONDING FIGURES IN ONE-, TWO-, THREE-, AND FOUR-SPACE.

FIG. 1.

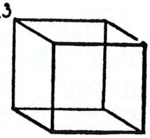
THE LINE: A 1-SPACE FIGURE GENERATED BY THE MOVEMENT OF A POINT; CONTAINING AN INFINITE NUMBER OF POINTS, AND 2 FORM ITS BOUNDARIES

FIG. 2.



THE SQUARE: A 2-SPACE FIGURE GENERATED BY THE MOVEMENT OF A LINE IN A DIRECTION PERPENDICULAR TO ITSELF TO A DISTANCE EQUAL TO ITS OWN LENGTH IT CONTAINS AN INFINITE NUMBER OF LINES, AND IS BOUNDED BY 4 LINES AND 4 POINTS.

FIG. 3



THE CUBE: A 3-SPACE FIGURE OR "SOLID." GENERATED BY THE MOVEMENT OF A SQUARE, IN A DIRECTION PERPENDICULAR TO ITS OWN PLANE, TO A DISTANCE EQUAL TO THE LENGTH OF THE SQUARE THE CUBE CONTAINS AN INFINITE NUMBER OF PLANES (SQUARES) AND IS BOUNDED BY 6 SURFACES, 12 LINES AND 8 POINTS

FIG 4

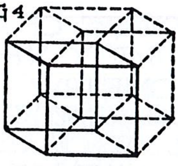
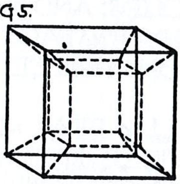


FIG 5.



THE TESSERACT, OR TETRA-HYPERCUBE: A 4-SPACE FIGURE GENERATED BY THE MOVEMENT OF A CUBE IN THE DIRECTION (TO US UNIMAGINABLE) OF THE 4TH DIMENSION. THIS MOVEMENT IS EXTENDED TO A DISTANCE EQUAL TO ONE EDGE OF THE CUBE AND ITS DIRECTION IS PERPENDICULAR TO ALL OUR 3 DIMENSIONS AS EACH OF THESE 3 IS PERPENDICULAR TO THE OTHERS. THE TESSERACT CONTAINS AN INFINITE NUMBER OF FINITE 3-SPACES (CUBES) AND IS BOUNDED BY 8 CUBES, 24 SQUARES, 32 LINES AND 16 POINTS.

NOTE: FIGURE 4 IS A SYMBOLIC REPRESENTATION ONLY—A SORT OF DIAGRAM—SUGGESTING SOME RELATIONS WE CAN PREDICATE OF THE TESSERACT. FIGURE 5 IS A REPRESENTATION DRAWN ON A DIFFERENT PRINCIPLE IN ORDER TO BRING OUT A DIFFERENT SET OF RELATIONS.

CORRELATIONS, IN FORM AND SPACE, OF SOME PROPERTIES OF ABSTRACT NUMBER

THE 3 LINEAR UNITS

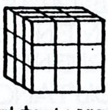
A NUMBER MULTIPLIED BY ITSELF GIVES THE SECOND POWER OF THAT NUMBER, COMMONLY CALLED ITS "SQUARE" BY REASON OF ITS CLOSE RELATION TO THE GEOMETRICAL SQUARE WHOSE SIDE CONTAINS THE GIVEN NUMBER OF UNITS OF LENGTH.

FIG. 1.

SIDE OF SQUARE 3 <u>LINEAR</u> UNITS. AREA OF SQUARE 9 <u>SQUARE</u> UNITS

THE SECOND POWER OF 3 IS 3 TIMES 3, OR 9. FIG 1 REPRESENTS A GEOMETRICAL SQUARE WHOSE SIDE IS 3 UNITS IN LENGTH, SAY 3 INCHES THE AREA OF THE SQUARE WILL OBVIOUSLY BE 9 SQUARE INCHES

FIG. 2



EDGE OF CUBE 3 <u>LINEAR</u> UNITS FACE OF CUBE 9 <u>SQUARE</u> UNITS VOLUME OF CUBE 27 <u>CUBIC</u> UNITS

LET US NOW BUILD UP FROM THE SQUARE TO A HEIGHT OF 3 INCHES THE CUBE REPRESENTED IN FIG 2. THE SOLID WILL OBVIOUSLY CONTAIN 3 X 3 X 3 = 27 <u>CUBIC</u> INCHES.

BY ANALOGY WITH THE GEOMETRICAL FIGURE THE NUMBER 27, THE 3RD POWER OF 3, IS CALLED IN ARITHMETIC THE "CUBE" OF 3

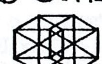
NOW THE 4TH, 5TH AND HIGHER POWERS OF A NUMBER ARE COMMONPLACES OF ARITHMETIC. WHAT DO SUCH HIGHER POWERS MEAN IN GEOMETRY?

WE CANNOT MAKE COMPLETE PHYSICAL REPRESENTATION OF 4-DIMENSIONAL SOLIDS IN OUR 3-SPACE, JUST AS WE CANNOT CONSTRUCT A CUBE IN A PLANE SURFACE BUT WE <u>CAN</u> MAKE DIAGRAMS OF HYPERSOLIDS, AND THE PROPERTIES OF MANY SUCH FIGURES IN HYPERSPACE ARE WELL KNOWN, HAVING BEEN DEMONSTRATED LIKE PROPOSITIONS IN EUCLID.

HYPERSPACE IS THUS <u>MATHEMATICALLY REAL</u>, AND THE MASTER MINDS OF SCIENCE CONSIDER IT TO BE <u>PHYSICALLY POSSIBLE</u> (LORD KELVIN AND OTHERS)

A 2-SPACE UNIT

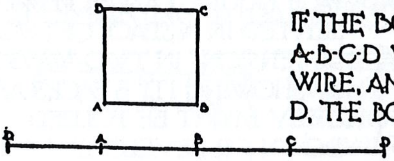
A 3-SPACE UNIT



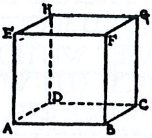
A 4-SPACE UNIT

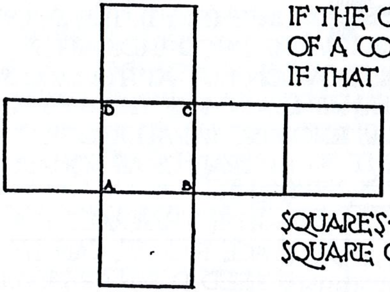
PLATE 2

THE DEVELOPMENT OF A UNIT OF 2, 3, AND 4 SPACE INTO THE NEXT LOWER SPACE AND THEIR EXPRESSION IN AND BY MEANS OF UNITS OF THOSE LOWER SPACES

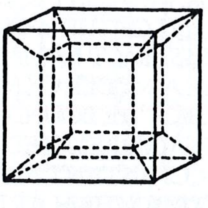


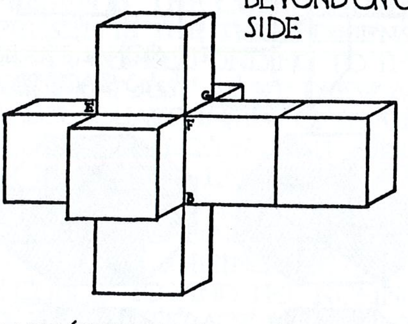
IF THE BOUNDING LINES OF THE SQUARE A-B-C-D WERE MADE OF A CONTINUOUS WIRE, AND IF THAT WIRE WERE CUT AT D, THE BOUNDARY COULD THEN BE BENT DOWN INTO LINE WITH A-B FORMING A ONE-DIMENSIONAL FIGURE OF FOUR LINEAR UNITS—THE ORIGINAL LINEAL UNIT A-B HAVING ONE LINEAL UNIT AT EACH END OF IT AND AN EXTRA ONE BEYOND AT ONE END





IF THE CUBE A-B-C-D—G WERE MADE OF A CONTINUOUS SHEET OF TIN AND IF THAT SHEET WERE CUT ALONG CERTAIN LINES FORMED BY INTERSECTING FACES, THE WHOLE COULD BE FOLDED DOWN TO FORM A TWO DIMENSIONAL FIGURE OF SIX SQUARES—THE SQUARE A-B-C-D HAVING A SQUARE ON EACH SIDE OF IT AND ONE BEYOND ON ONE SIDE





SIMILARLY IF THE TESSERACT (REPRESENTED BY THE DIAGRAM) WERE MADE OF SOLID WOOD AS TO ITS BOUNDING CUBES AND IF THIS WOOD WERE CUT THROUGH THE APPROPRIATE PLANES, THE CUBES COULD, BY ANALOGY, BE FOLDED DOWN TO FORM A THREE DIMENSIONAL FIGURE OF EIGHT CUBES

CORRESPONDING DEVELOPMENTS AND PROJECTIONS OF A CUBE AND OF A TESSERACT IN LOWER SPACES

FIG 1

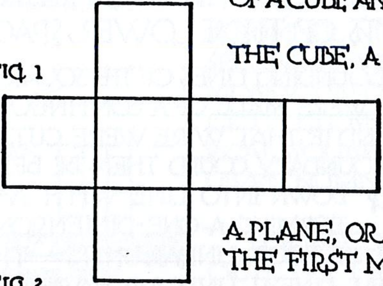


FIG 2

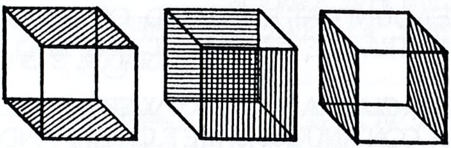


FIG 3

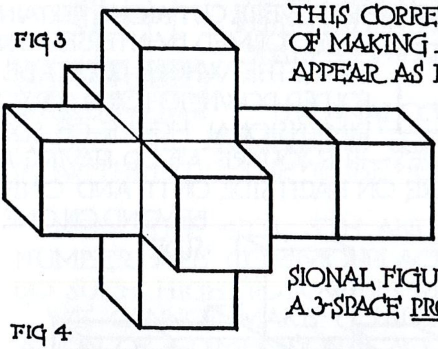
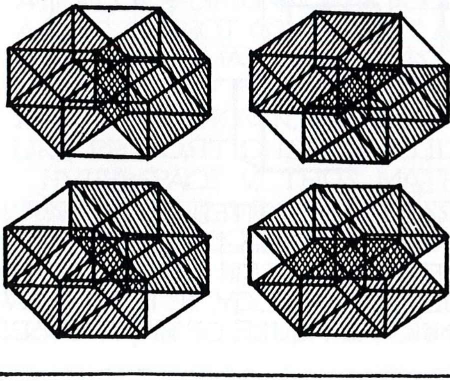


FIG 4



THE CUBE, A 3-SPACE FIGURE, CAN BE REPRESENTED IN A SPACE OF TWO DIMENSIONS IN TWO WAYS: BY SHOWING ITS 6 FACES AS THEY MIGHT BE FOLDED DOWN OR DEVELOPED UPON A PLANE, OR BY A PROJECTIONAL DRAWING. THE FIRST METHOD WOULD SHOW THE SQUARES WITHOUT DISTORTION, BUT WOULD GIVE SMALL IDEA OF THEIR CORRELATION INTO A UNIT (FIG 1). THE SECOND METHOD INDICATES THIS CORRELATION, BUT AT THE EXPENSE OF MAKING AT LEAST 4 OF THE 6 FACES APPEAR AS RHOMBIC PARALLELOGRAMS INSTEAD OF AS SQUARES (FIG 2).

THE TESSERACT, A 4-SPACE FIGURE, CAN BE FOLDED DOWN, OR DEVELOPED INTO A 3-DIMENSIONAL FIGURE—OR WE MIGHT CONSTRUCT A 3-SPACE PROJECTION OF THE 4-SPACE TESSERACT BY MAKING, SAY, A MODEL OF WIRE, IN WHICH THE BOUNDARIES ARE CORRELATED INTO A CONSISTENT FIGURE, BUT IN WHICH AT LEAST 6 OF THE 8 BOUNDING CUBES ARE DISTORTED INTO PARALLELOPIPEDS.

THE FOURFOLD REPETITION OF FIG 4 IS FOR THE PURPOSE OF EXHIBITING THE 8 BOUNDING CUBES OF THE TESSERACT

PLATE 4