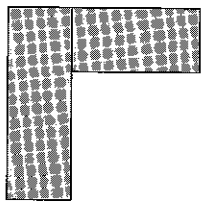
EXPLORATION 9.5 Paper Folding

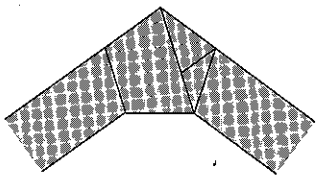
Begin with square sheets of paper.

PART 1: Making geometric figures with paper folding

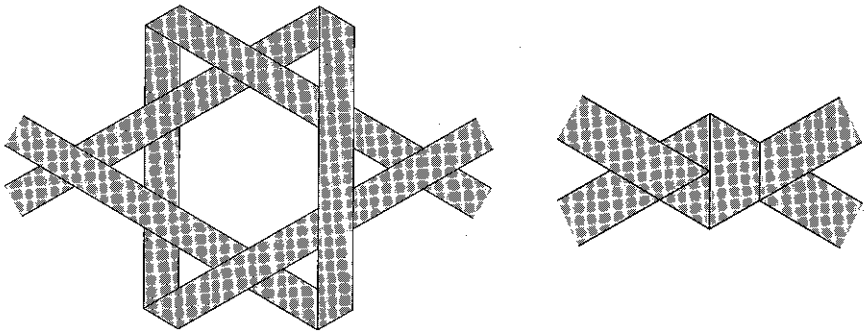
1. ***Square*** Cut two strips of paper of equal width. Fold each strip onto itself. Insert one strip inside the other so that they interlock, as shown at the right. Cut off the excess paper, and you have a square (actually four squares if you unfold what remains and cut at the folds). Why are these figures squares?



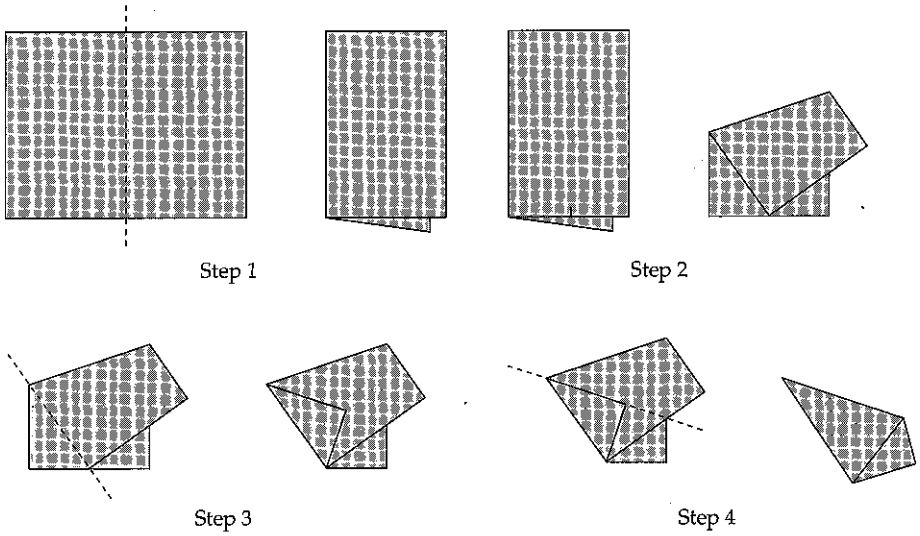
2. ***Pentagon*** Cut a long, thin strip of paper. (This exercise will work with a thin strip cut from $8\frac{1}{2}$ by 11 inch notebook paper). Tie a regular overhand knot with the paper and carefully tighten the knot until your paper looks like the figure at the right. You may have to play with it for a bit. When you cut off the two excess pieces, you have a regular pentagon. What can you see from the folds that can help you to understand why the resulting figure is a regular pentagon?



3. ***Hexagon*** Cut two thin strips of paper. Again, this will work with strips cut from $8\frac{1}{2}$ by 11 inch notebook paper. Tie a square knot with the two pieces of paper. The figure at the left below shows the parts of each strip that are above and below the other strip. Be patient—it takes a couple readings to figure this out. Carefully tighten the knot; as you begin to tighten the knot, your strips should look like the figure at the right. (*Hint*: Make sure that the left and right end pieces are on top of each other as shown in the figure at the left.) What can you see from the folds that can help you to understand why the resulting figure is a regular hexagon?



4. ***A family of fives***
   - **a.** Take a sheet of notebook paper and fold it in half as shown in Step 1.
   - **b.** Find the midpoint of the bottom edge by folding. Bring the top left corner of the paper to the midpoint and fold, as shown in Step 2.
   - **c.** Fold the bottom left corner across the fold line as shown in Step 3.
   - **d.** Finally, fold the top edge across the fold line as shown in Step 4. Your result should look like the righthand figure in Step 4.

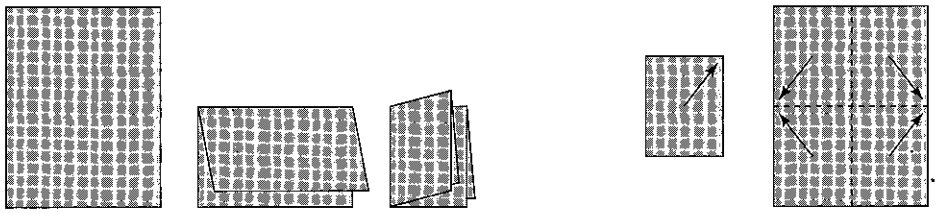


e. Cut off the tip at an angle and then open up your paper.
f. Repeat steps (a) through (e), making cuts of various sizes and angles at the tip.
g. Present your results. Explain why each cut produces the kind of figure that it produces.

PART 2: Predicting what happens when you unfold the paper[1]

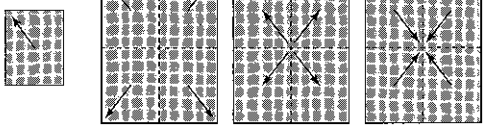
1. In the example shown below, a piece of paper is folded twice, and then an arrow is drawn in the upper right corner. This is a special type of paper: When the arrow is drawn, its image appears on each of the three layers below. When the paper is unfolded, the four arrows appear as shown.

Example:

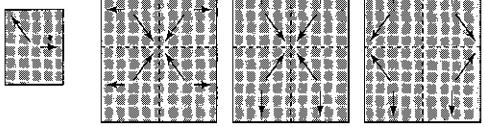


In each part, select the diagram (1, 2, or 3) that shows what the given paper will look like when it is unfolded. Briefly summarize your thinking process.

a.



b.



[1]"Which Way Will the Arrow Point?" from Seeing Shapes by Ernest R. Ranucci. In *Geometry and Visualization* by Mathematics Resource Project. © 1977. Creative Publications, Inc.

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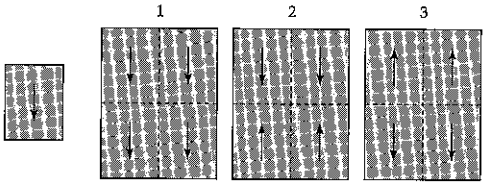
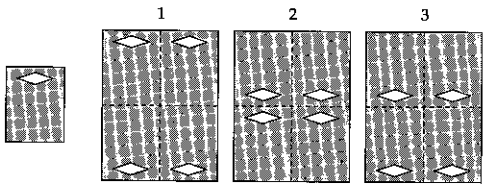
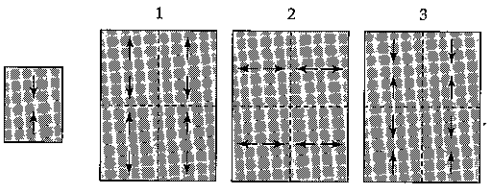
c.



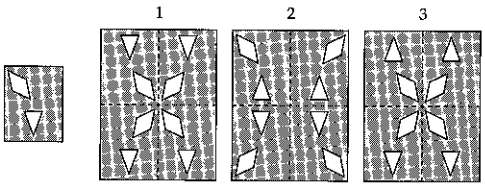
d.



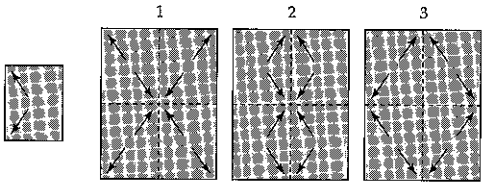
e.



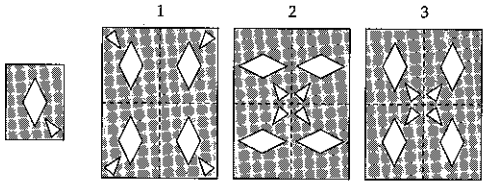
f.



g.



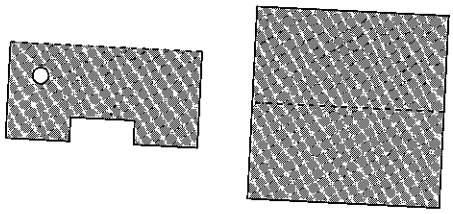
h.



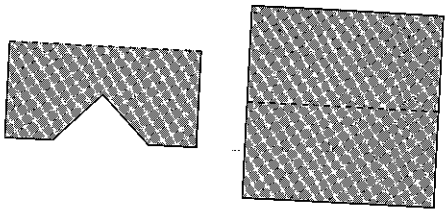
2. Use the diagrams on p. 277. In each part, a square piece of paper is folded and cut as shown in the left figure. In parts (a) through (d), the square paper is folded in half once. In parts (e) through (h), the square piece of paper is folded in half twice. The dotted lines represent the fold lines. In each case, use the right figure to draw what the design will look like when the paper is unfolded. Briefly summarize your thinking process.

Figures for EXPLORATION 9.5, PART 2: What happens when you unfold the paper?

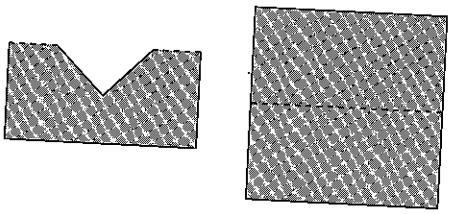
a.



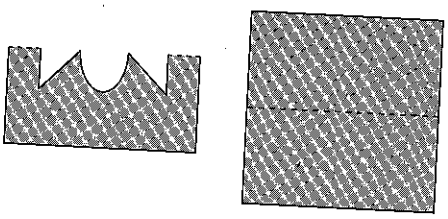
b.



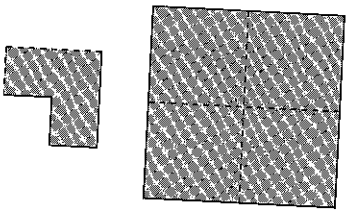
c.



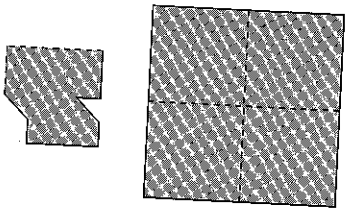
d.



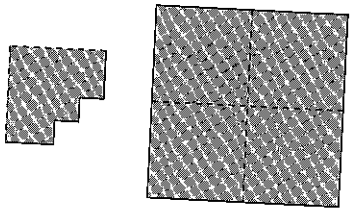
e.



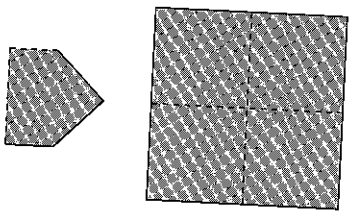
f.



g.



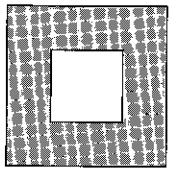
h.

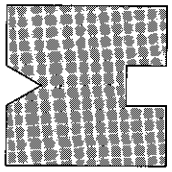


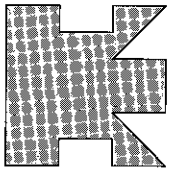
PART 3: Making copies of designs

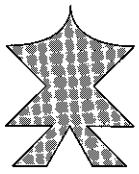
For each of the questions in this part, whether you produce the design on your first attempt or after several attempts, turn in your solution and a summary of your thinking process. In cases of more than one attempt, include your "failures" and a description of what you learned from each "failure." Many students say they learned a lot from their wrong answers.

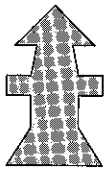
1. Fold a square piece of paper in half. Determine the cut(s) needed to produce each design when the paper is unfolded.

a. 

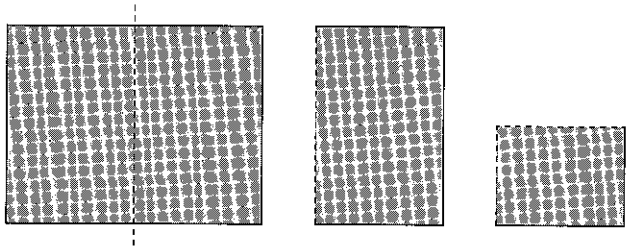
b. 

c. 

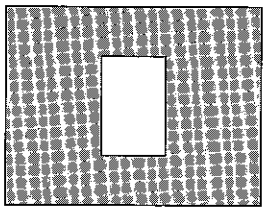
d. 

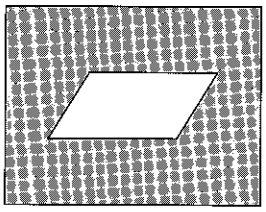
e. 

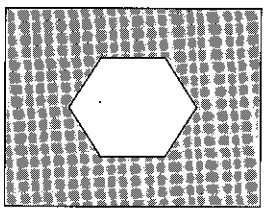
2. Fold an $8\frac{1}{2}$ by 11 inch sheet of paper in half and then in half again, as shown.

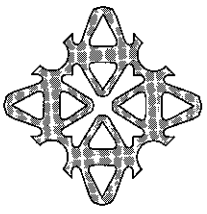


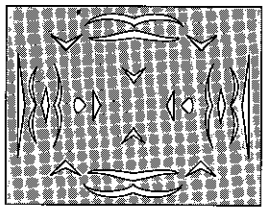
Determine the cut(s) needed to produce each design when the paper is unfolded.

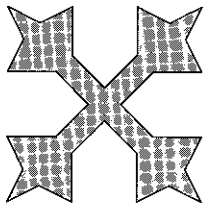
a. 

b. 

c. 

d. 

e. 

f. 

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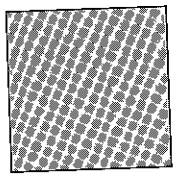
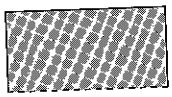
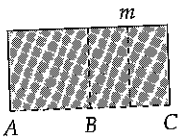
3. Perform the following steps on a square piece of paper. If all goes as planned, you should end up with the paper containing, when unfolded, six 60-degree angles.



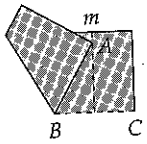
Step 1.
Begin with a square sheet of paper.



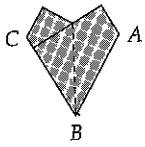
Step 2.
Fold the sheet in half.



Step 3.
Fold this sheet in half vertically. Fold the right half in half, forming line m.



Step 4.
Fold the paper at point B by bringing point A to line m. This guarantees that angle $ABC = 60°$. Why?

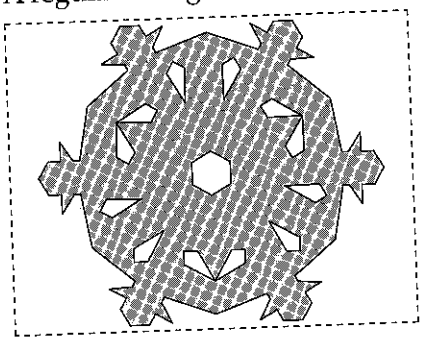


Step 5.
Fold the paper along AB, bringing point C over so that it lies on top of the left edge of the paper.

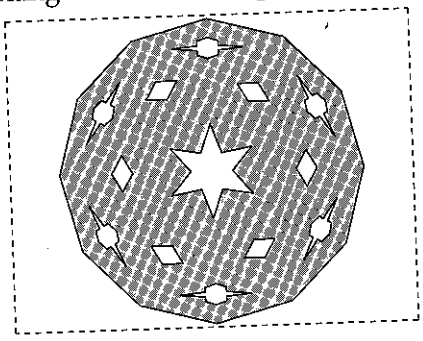
Describe how you would cut the folded paper from Step 5 to make each of the following.

a. A regular hexagon **b.** An equilateral triangle **c.** A six-pointed star

d.



e.



f. Find a picture of a snowflake on the Internet or in a book and make a paper replica of that snowflake.