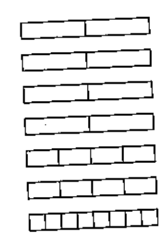
SECTION 5.2 Exploring Fractions and Rational Numbers

Researchers have told us for years that students' understanding of rational numbers is poor. This is unfortunate because the concept of rational numbers is one of the "big ideas" in elementary school mathematics. The fraction explorations here have been designed to give you an opportunity to work with the fundamental concepts related to fractions. Just as we discussed different decompositions of whole numbers in Chapters 3 and 4, a key to understanding fractions is to decompose them. That is, the notion of parts and wholes connects to our compositions and decompositions with whole numbers. We shall investigate various decompositions, which in turn will deepen your understanding of the different fraction contexts and the relationship between the numerator and the denominator.

EXPLORATION 5.5 Egyptian Fractions and Sub Sandwiches

When we ask children to explore, instead of just showing them how to get answers, their natural solution paths often mirror strategies developed over the centuries in different parts of the world. If you did the exploration on Alphabitia in Chapter 3, you saw this first-hand.



The problem that prompted this exploration is this: How can 8 kids share 7 sub sandwiches? One solution path, shown at the right, is to figure out the biggest reasonable piece that all can share and then move to smaller and smaller pieces. For example, first give everyone 1/2 a sandwich. Then give everyone 1/4 of a sandwich. Now you have one sandwich left, so give everyone 1/8 of a sandwich. Everyone gets 1/2 + 1/4 + 1/8. This process is very similar to how the Egyptians worked with fractions. As stated in the text, the Egyptians, with the exception of 2/3, used only unit fractions. Thus, for 7/8, they wrote 1/2 + 1/4 + 1/8. It turns out that any fraction can be decomposed into some combination of different unit fractions.

This exploration asks you to think like an Egyptian and decide how big a share each person gets in the following division problems. A valid answer consists of a sum of unit fractions, and all of the fractions need to be different; for example, 7/10 = 1/2 + 1/10 + 1/10 is not valid. Look for patterns and strategies that will enable you to be more effective.

Notes: I have omitted problems where the answer is immediately a unit fraction or 2/3, such as 6 children sharing 2 subs (1/3), 6 children sharing 3 subs (1/2), and 6 children sharing 4 subs (2/3). In most of the cases, the solution will involve the sum of 2 fractions. In the other cases, it will involve the sum of 3 fractions.

1. How would 4 children share 3 sub sandwiches?
2. How would 5 children share 2 sub sandwiches?
3. How would 5 children share 3 sub sandwiches?
4. How would 5 children share 4 sub sandwiches?
5. How would 6 children share 5 sub sandwiches?
6. How would 7 children share 2 sub sandwiches?
7. How would 7 children share 3 sub sandwiches?

8. How would 7 children share 4 sub sandwiches?
9. How would 8 children share 3 sub sandwiches?
10. How would 8 children share 5 sub sandwiches?
11. How would 12 children share 5 sub sandwiches?
12. How would 12 children share 7 sub sandwiches?
13. Describe any shortcuts or faster strategies that you developed while doing these problems.
14. Describe any growth in your understanding of fractions (such as your understanding of equivalent fractions, factors, or LCM) that resulted from doing these problems.