And now for something completely different. Good news - we're done with the most abstract part of the course, just a few things remaining here. We have a quick trip through a long and complicated topic - but the great thing about it being quick is - it can't get too complicated.

Remember when you did anti-derivatives when you were just starting integration? You started with very simple examples like, find y such that its derivative is $2x$. But, wait, that's our idea, it's easy to miss, but it's our idea. That, actually, is an equation. It looks like this: $y' = 2x$. It's an equation with a derivative in it. We call this a "differential equation". A solution to this equation is a function $y(x)$. (1) What is one solution to this differential equation? (2) Remember, there's not only one, what is another one? (3) What is a solution that has $y(0) = \pi$? That is a differential equation with an initial condition, i.e. a starting value. We'll be studying these differential equations for exactly one class week, and then we'll move on.

Wait, you say ... aren't these just integrals asked in a strange way? Well, kinda. If our differential equations is just $y' = f(x)$, then the solution is $y = \int f(x)dx$. But, and what is much more interesting, what if y' is not given in terms of x? Here's an example that you *should* be able to find answers for, but *not* by integrating. (4) What is a solution to $y' = y$? It works differently this time. There is more than one answer, but not in the same way. (5) What is another solution? (6) What is a solution with $y(0) = 4$? We're back to the beginning suddenly. At this very moment the only way we have to solve differential equations like this one is if we happen to guess correctly.

Let's try guessing a few more here to prepare for lab next class. (7) What is a solution to $y' = 2y$? (8) What is a solution with $y(0) = 6$? (9) What is a solution to $y' = -y$? (10) What is a solution with $y(0) = 7$? That should get a good start on the before the lab.

When you first started to learn about solving equations (in some kind of pre-algebra class), you quickly saw that *checking* equations is much easier than solving them. Let's do one of those and then leave it behind, since ... it's dull. Consider the differential equation $y' = x + y$ which looks very innocent. It turns out that solving it is fancier than we will do in our week, but we can easily check a solution. (11) Check that $y = e^x - x - 1$ is *a* solution to this differential equation.

Ok, great. We have one way to solve differential equations so far - if they are $y' = f(x)$, and only if we *can* integrate. We will talk about another way a later day, but can we *analyse* them without solving them? This is .. different, but interesting. Remember, the derivative, y', tells the slope of the function at the point. So, one way to interpret our differential equations is that they tell us the slopes at each of the points. We can graph these. The graph is curious. It's a bunch of little pieces of slopes for different x and y values. And, I hope you saw when you opened this worksheet, there's a link next to it for some software help. Please go back to the syllabus and open that link. It is used to graph *slope fields*. (12) Use the linked software to look at a slope field for $y' = x + y$. (I think it should load with that one.) (13) Pick some values for (x, y) and check if the slope there makes sense. Maybe $(1, -1)$, or $(-1, 2)$. We can sketch solutions to differential equations this way without computing by just following the slopes - making a curve that has the slopes indicated by the slope field. (14) Notice that you can also select a point and the software will follow the slopes for you. (15) Experiment with the software. (16) Make notes of anything interesting that you find. What did you find? We're just getting started today. We'll experiment more in class.

Next class in lab we will look at a way to compute numeric values for the solution to a differential equation. Instead of getting a function, we will get a table. To preview that, say these one after the other enough times so that you remember they are pronounce the same "oiler Euler". In fact the numeric way uses the slopes. I will talk about it as much as we have time for in class.