222 §1.1 Areas between Curves

When you started integration you worked with $\int_a^b f(x)dx$ as the area under the curve $f(x)$ and over the x-axis between the vertical lines $x = a$ and $x = b$. Furthermore if $f(x) < 0$ on the interval, then $\int_a^b f(x)dx$ computes the area *below* the x-axis and *over* the curve, but with a negative sign, as if area below were counted as negative. (1) Make a graph of $x^2 + 6$ and $5 - 3x^6$ on the interval $[-1, 1]$. Notice that the first graph is always over the second. (2) How do you compute the area under the first graph over that interval? (3) How do you compute the area under the second graph over that interval? (4) Because the second is always under you may subtract to find the area between the curves. Since the limits are the same we may combine them into one integral which could be written as $\int_{-1}^{1}(x^2 + 6) - (5 - 3x^6)dx$. In general we may find the area between two curves by subtracting the lower from the upper and integrating.

Sometimes we're not told the limits for problems like these. For example, find the area enclosed between the parabolas $f(x) = -2x^2 + 4$ and $g(x) = x^2 - 9x + 10$. What does "enclosed between" mean? (5) Make a graph to find out. See that they cross to hold in area. This also gives us an idea where to find the limits. We will need to know where the two curves intersect. That's a familiar pre-calculus task. (6) How do you do that? Do so, what do you find. for the intersections points? Does that match what you see on the graph? (I could call that 3 questions, but it's all tied together.). I trust that you found the intersections at $x = 1$ and $x = 2$. These give us our bounds of integration. (7) Look at the graph to see which one is on top between those limits. (8) Set up the integral that will give the enclosed area. If you like, compute the numeric value.

In the last example, the graphs intersected twice and went along their way. Here's an example where they cross many times. Find the area between $f(x) = \sin x$ and $g(x) = \cos x$ over $[-\pi/2, 3\pi]$. (9) Look at a graph and count the intersections. Be sure to remember which curve is which. Because we have limits this time, we also need to include the sections that they enclose on the far left and right. Altogether I hope that you see there are 4 sections. (10) Write each of the four integrals needed for this area. They don't combine together in any nice way, only to add together at the end. Compute to one numeric value if you like. Most importantly, check to be sure that you see that what you've written would produce one numeric value if computed.

If we work in terms of x instead of y we use the same method to find the area between curves horizontally, with one on the left and one the right. Your graphing machines may not be as fluid with this as I hope you are. For our last example, we will switch the roles of x and y but basically the rest is the same ... compute the area enclosed by the curves $x = y^2 - y - 4$ and $x = -y^2 + 3y + 12$. Since this is so similar to the ones before, the steps will be about the same. (11) Where do the curves intersect? (12) Which one is bigger, so will come first in the subtraction in the appropriate interval? (13) Write the integral that is needed to compute this area.

We will discuss all of this in class. Please bring questions.