Lab 17: Patterns of Integrals

Goals

- To become familiar with the types of functions that are the antiderivatives of given families of functions.
- To get practice drawing conclusions from patterns of answers.

In the Lab

In this lab you will use a computer to calculate the antiderivative of several functions. Each problem concerns a family of related functions. You are to develop an intuition about the form of the antiderivative of each of these families.

1. a. Determine antiderivatives of the following rational functions.

   i. $\displaystyle\int \frac{1}{(x+1)(x+2)}\,dx$

   ii. $\displaystyle\int \frac{1}{(x+2)(x-7)}\,dx$

   iii. $\displaystyle\int \frac{1}{x(x-2)}\,dx$

   iv. $\displaystyle\int \frac{1}{(x-3)^2}\,dx$

   b. Based on the pattern of your answers to parts *i–iii*, determine an antiderivative for $\dfrac{1}{(x+p)(x+q)}$ where $p \neq q$. What changes do you need to make in your answer in the case where $p = q$?

2. a. Determine antiderivatives of the following powers of x times the exponential function.

   i. $\displaystyle\int x\,e^x\,dx$

   ii. $\displaystyle\int x^2 e^x\,dx$

   iii. $\displaystyle\int x^5 e^x\,dx$

iv. $\int x^7 e^x \, dx$

b. From your answers in part a, determine $\int x^n e^x \, dx$.

3. a. Determine antiderivatives of the following powers of x times the logarithm function.

i. $\int \ln x \, dx$

ii. $\int x \ln x \, dx$

iii. $\int x^3 \ln x \, dx$

iv. $\int x^6 \ln x \, dx$

b. Based on your answers in part a, determine an antiderivative of $x^n \ln x$.

4. a. Determine antiderivatives of the following powers of the cosine function.

i. $\int \cos x \, dx$

ii. $\int \cos^3 x \, dx$

iii. $\int \cos^5 x \, dx$

iv. $\int \cos^2 x \, dx$

b. Based on your answers to parts *i–iii*, determine the form of the antiderivative of $\cos^n x$, where n is a positive odd integer. (When giving your answer, do not worry about the value of the coefficients.) Note that the pattern is different when n is an even integer.

5. a. Determine antiderivatives of the following products of sine functions.

i. $\int \sin x \sin 2x \, dx$

ii. $\int \sin 2x \sin 5x \, dx$

iii. $\int \sin 6x \sin 2x \, dx$

b. Using your answers to part a, determine the form of $\int \sin ax \sin bx \, dx$.

Further Exploration

6. Refer to your work for Problem 1. Use partial fractions to prove that your formula for $\int \frac{1}{(x+p)(x+q)} \, dx$ is correct in the case where $p \neq q$.

7. Refer to your work for Problem 3.

a. Does your formula for $\int x^n \ln x \, dx$ continue to work for negative values of n? Be careful about the case $n = -1$. If necessary, modify your formula.

b. Use integration by parts to prove that your formula for $\int x^n \ln x \, dx$ is correct.