## 222 Assignment 3 Solutions

§2.4 20. $\int \frac{2x^2+4x+22}{x^2+2x+10}dx$. I'm apparently not very good at avoiding problems with solutions in the text. Well, as they said the first step is to divide to get the numerator degree lower: $(2x^2+4x+22)\div(x^2+2x+10) = 2+\frac{2}{x^2+2x+10} = 2+\frac{2}{x^2+2x+1+9} = 2+\frac{2}{(x+1)^2+9} = 2+\frac{2}{9(\{[x+1)/3]^2+1\}}$. Now we've done enough manipulations to integrate, $\int \frac{2x^2+4x+22}{x^2+2x+10}dx = \int 2+\frac{2}{9(\{[x+1)/3]^2+1\}}dx = 2x+\frac{2}{3}\tan^{-1}(\frac{x+1}{3})+C$. Notice that we compensate for the 3 in the denominator by multiplying by 3, hence canceling one from 9.

§2.4 27. $\int \frac{x^2}{x^3-x^2+4x-4}dx$. I was hoping it factored to $(x-1)(x-2)(x+2)$, but not quite, alas. Apparently it's $(x-1)(x^2+4)$. So we set out partial fractions $\frac{x^2}{x^3-x^2+4x-4} = \frac{A}{x-1}+\frac{Bx+C}{x^2+4}$. We clear fractions and find $x^2 = A(x^2+4)+(Bx+C)(x-1)$. $x=1$ produces $1=5A$, so $A=\frac{1}{5}$. Gathering equations by power (mixing my methods here), squares: $1=A+B$, $0=C-B$ (oh, so $C=B$, that's nice), and $0=4A-C$ (and so $C=4A$ also pretty workable). So, altogether we have $A=\frac{1}{5}$, $C=\frac{4}{5}$, $B=\frac{4}{5}$. Reassembling: $\int \frac{x^2}{x^3-x^2+4x-4}dx = \int \frac{1/5}{x-1}+\frac{4/5x}{x^2+4}+\frac{4/5}{x^2+4}dx = \frac{1}{5}\ln(x-1)+\frac{2}{5}\ln(x^2+4)+\frac{2}{5}\tan^{-1}(\frac{x}{2})+C$.

§2.5 35. $\int \frac{dx}{1+\sin x}$. I want to get that sum out of the denominator, so I'll multiply numerator and denominator by $(1-\sin x)$ to get $\cos^2 x$ and something we can work with. Let's see: $\int \frac{1-\sin x}{1-\sin x}\frac{dx}{1+\sin x} = \int \frac{1-\sin x}{\cos^2 x}dx = \int \sec^2 x - \frac{\sin x}{\cos^2 x}dx$. The first is nice, for the second, let $u=\cos x$, this then gives $\int \frac{dx}{1+\sin x} = \tan x - 1/(\cos x)+C = \tan x - \sec x + C$. (for some detail: there are 3 negatives that combine together to give the final $-\sec x$, one is from $\int \sec^2 x - \frac{\sin x}{\cos^2 x}dx$, one is from $du = -\sin x dx$ and one is from $\int \frac{du}{u^2} = -\frac{1}{u}+C$. According to the old rule, three negatives make a negative.)

§2.6 18 (use *Simpson's* Rule). $\int_0^1 \frac{dx}{1+x^2} = [\tan^{-1}x|_0^1 = \tan^{-1}(1) = \frac{\pi}{4}$. Simpson's rule with $n=4$, just gives us two parabolas, 14241, so we add $\frac{1}{12}\left(\frac{1}{1+0^2}+\frac{4}{1+(1/4)^2}+\frac{2}{1+(1/2)^2}+\frac{4}{1+(3/4)^2}+\frac{1}{1+1^2}\right) = \frac{8011}{10200}$ which is a pretty good approximation to $\frac{\pi}{4}$.

§2.6 45. Tables are where numerical integration excels. We have no way of computing anti-derivatives from a table, but numerical integration does just fine. Trapezoid gives us $\frac{100}{2}(125+2(125+120+112+90+95+88+75+35)+0) = 89250$ m$^2$.

§2.7 40 $\int_{-27}^1 \frac{dx}{x^{2/3}}$. The problem is at $x=0$ so we divide into two integrals: $\int_{-27}^1 \frac{dx}{x^{2/3}} = \int_{-27}^0 \frac{dx}{x^{2/3}} + \int_0^1 \frac{dx}{x^{2/3}}$. Now since the function is not defined for zero, we'll need to sneak up to it on both sides, using two separate limits. This gives us: $= \lim_{t\to 0^-}\int_{-27}^t \frac{dx}{x^{2/3}} + \lim_{s\to 0^+}\int_s^1 \frac{dx}{x^{2/3}}$. Fortunately the integrals are easy and the same, therefore: $= \lim_{t\to 0^-}[3x^{1/3}|_{-27}^t + \lim_{s\to 0^+}[3x^{1/3}|_s^1 = \lim_{t\to 0^-} 3t^{1/3} - (-9) + 3 - \lim_{s\to 0^+} 3s^{1/3} = 0-(-9)+3-0 = 12$.

§2.7 46 $\int_1^4 \frac{dx}{\sqrt{x^2-1}}$ The problem is at $x=1$, so we need a limit to get there: $\lim_{t\to 1^+}\int_t^4 \frac{dx}{\sqrt{x^2-1}}$. This time we've got more work to do on the integral. I'm going to focus on the integral for a while then come back to the limits (in both meanings): $\int \frac{dx}{\sqrt{x^2-1}}$. Let $x=\sec\theta$ so that we can undo the square root. $\sqrt{x^2-1} = \sqrt{\sec^2\theta-1} = \sqrt{\tan^2\theta} = \tan\theta$. For that to work we need $dx = \sec\theta\tan\theta d\theta$. So, $\int \frac{dx}{\sqrt{x^2-1}} = \int \frac{\sec\theta\tan\theta d\theta}{\tan\theta} = \int \sec\theta d\theta = \ln(\sec\theta+\tan\theta)+C = \ln(x+\sqrt{x^2-1})+C$. Ok, now let's bring back those limits, $\lim_{t\to 1^+}\int_t^4 \frac{dx}{\sqrt{x^2-1}} = \lim_{t\to 1^+}[\ln(x+\sqrt{x^2-1})|_t^4 = \ln(4+\sqrt{15}) - \lim_{t\to 1^+}[\ln(t+\sqrt{t^2-1}) = \ln(4+\sqrt{15}) - \ln 1 = \ln(4+\sqrt{15})$

Wow, solutions in one page. Probably good given all we've got happening.