I originally created this document as a place to demonstrate how to make tables in LaTeX, since that was something that came up in a recent meeting. But when we wanted a place to write a formal proof using the contrapositive, this document was convenient, and it made a good opportunity to illustrate how you can divide a LaTeX document into sections, too.

1 The Contrapositive

For an example of proofs using the contrapositive, consider this claim:

Theorem 1. *For all real numbers x and all integers $n \neq 0$, if x is an irrational number, then nx is also irrational.*

Proof. We will show that for all real numbers x and all integers $n \neq 0$, nx is irrational if x is by proving the contrapositive. The contrapositive of the theorem is "for all real numbers x and all integers $n \neq 0$, if nx is rational than x is rational." To prove this, we assume n is an integer and x is a real number such that nx is rational. From the definition of rational numbers, this means $nx = \frac{a}{b}$ for integers a and $b \neq 0$. Dividing by n then yields

$$x = \frac{a}{nb}$$

Since integers are closed under multiplication, and neither n nor b are 0, $\frac{a}{nb}$ is rational. This proves that for all real numbers x and all integers $n \neq 0$, if nx is rational then x is rational. It thus also proves the contrapositive, that for all real numbers x and all integers $n \neq 0$, if x is an irrational number, then nx is also irrational. □

2 Tables

This section presents an example of building simple tables in LaTeX. For example, here's a truth table for "and."

P	Q	$P \wedge Q$
T	T	T
T	F	F
F	T	F
F	F	F

3 Biconditionals

Theorem 2. *For all integers n, n is even if and only if $n+1$ is odd.*

Proof. We will prove that for all integers n, n is even if and only if $n+1$ is odd. We prove each direction separately.

We start by showing that if n is even, then $n+1$ is odd. From the definition of even numbers, $n = 2m$ for some integer m. Therefore $n+1 = 2m+1$, which meets the definition of an odd integer.

Next we show that if $n + 1$ is odd, then n is even. Since $n + 1$ is odd $n + 1 = 2m + 1$ for some integer m, and thus $n = 2m$. Since m is an integer, this n meets the definition of an even integer.

We have now shown that if n is even, then $n+1$ is odd, and that if $n+1$ is odd, then n is even. Together, these statements establish that n is even if and only if $n+1$ is odd. $\square$