In the following, we juxtapose two proofs about sets, one that uses an element chasing proof style and the other using set algebra. We start with the element chasing proof. . . .

Theorem 1. *If A and B are subsets of some universal set, then $A-B = A \cap B^{\mathrm{C}}$.*

Proof. We will show that if A and B are subsets of some universal set, then $A - B = A \cap B^{\mathrm{C}}$ by showing that every element of $A - B$ is also in $A \cap B^{\mathrm{C}}$ and vice versa.

To show that every element of $A - B$ is also in $A \cap B^{\mathrm{C}}$, let x be an element of $A - B$. From the definition of set difference, x must be in A and not in B. Since x is not in B, x is in B^{C}. Now, since x is in A and in B^{C}, x is in $A \cap B^{\mathrm{C}}$.

Next, to show that every element of $A \cap B^{\mathrm{C}}$ is also in $A - B$, let y be an element of $A \cap B^{\mathrm{C}}$. From the definitions of intersection and complement, y must be in A and not in B. Therefore, by the definition of set difference, y is in $A-B$.

We have now shown that every element of $A - B$ is also in $A \cap B^{\mathrm{C}}$ and vice versa, thereby establishing that the sets are equal. This completes the proof that if A and B are subsets of some universal set, then $A - B = A \cap B^{\mathrm{C}}$. □

Next, we turn to an example of an algebraic proof about sets.

Theorem 2. *If A, B, and C are subsets of some universal set, then $(A \cap B) - C = (A - C) \cap (B - C)$.*

Proof. We show that if A, B, and C are subsets of some universal set, then $(A \cap B) - C = (A - C) \cap (B - C)$ by using set algebra. We begin by rewriting set difference in terms of intersection, and then regroup the resulting expression:

$$\begin{aligned}
(A \cap B) - C &= (A \cap B) \cap C^{\mathrm{C}} \\
&= A \cap B \cap C^{\mathrm{C}} \cap C^{\mathrm{C}} \\
&= (A \cap C^{\mathrm{C}}) \cap (B \cap C^{\mathrm{C}}) \\
&= (A - C) \cap (B - C)
\end{aligned}$$

We have thus shown that if A, B, and C are subsets of some universal set, then $(A \cap B) - C = (A - C) \cap (B - C)$. □