Proofs of existential statements often boil down to finding an example that makes the predicate true. Sometimes these proofs look a bit silly as formal proofs, but they can still be written in that form. For example. . . .

Theorem 1. *There exists a real number x such that $3x^2 + 1 = 13$.*

Proof. We will show that there exists a real number x such that $3x^2 + 1 = 13$ by solving for such a value:

$$\begin{aligned} 3x^2 + 1 = 13 &\rightarrow 3x^2 = 12 \\ &\rightarrow x^2 = 4 \\ &\rightarrow x = \pm 2 \end{aligned}$$

We have thus found that 2 and -2 are real numbers that satisfy $3x^2 + 1 = 13$. Therefore we see that there exists a real number x such that $3x^2 + 1 = 13$. □

Proofs about universal quantifiers, on the other hand, tend to look more like what we're used to. For example. . . .

Theorem 2. *Every even integer can be written in the form $n + 2$ where n is another even integer.*

Proof. We assume that m is an even integer, and will show that $m = n + 2$ for some integer n, thereby establishing that every even integer can be written in the form $n + 2$ where n is another even integer. From the definition of even integer, $m = 2b$ for some integer b. Letting $n = m - 2$, we see that $m = n + 2$, and so we only need to show that n is also even. Substituting $m = 2b$ into the definition of n gives

$$\begin{aligned} n &= m - 2 \\ &= 2b - 2 \\ &= 2(b - 1) \\ &= 2a \end{aligned}$$

where $a = b - 1$. Because the integers are closed under subtraction and b is an integer, so is a, and so n is even. We have now shown that m can be written in the form $n + 2$ for some even integer n, and thus that every even integer is of the form $n + 2$ where n is another even integer. □