Theorem 1. *If a, b and c are integers such that $a|b$ and $a|c$, then $a|(b+c)$.*

Proof. We assume that a, b and c are integers such that $a|b$ and $a|c$, and will show that $a|(b+c)$. Since $a|b$, the definition of divisibility means that $b = xa$ for some integer x. Similarly $c = ya$ for some integer y. Now

$$\begin{aligned} b + c &= xa + ya \\ &= (x + y)a \end{aligned}$$

So we have shown that $b+c$ is an integer multiple of a, since $x+y$ is an integer by closure under addition. Thus we have shown that if a, b and c are integers such that $a|b$ and $a|c$, then $a|(b+c)$. □