For an example of what a proof by cases might look like when fully written out, suppose $f(x)$ is defined by the following:

$$f(x) = \begin{cases} x^2 & \text{if } x < 0 \\ x & \text{if } 0 \leq x \leq 1 \\ 2x - 1 & \text{if } 1 < x \end{cases} \tag{1}$$

Then we have

Conjecture 1. *For all real numbers x, $f(x)$ as defined by (1) is greater than or equal to* 0.

Proof. We will prove that $f(x) \geq 0$ for all real numbers x by analyzing each case in the definition in turn.

For the first case, consider $x < 0$. Then $f(x) = x^2$, which is greater than or equal to 0 for all values of x.

For the second case, consider $0 \leq x \leq 1$. In this case $f(x) = x$ which is greater than or equal to 0 from the constraint $0 \leq x \leq 1$.

Finally, consider $1 < x$. Here, $f(x) = 2x - 1$ which is an increasing function, i.e., $f(x) > f(1)$ for all $x > 1$. Since $f(1) = 1 \geq 0$, we see that $f(x) \geq 0$ for all $x > 1$.

We have now shown that $f(x) \geq 0$ for all cases in the definition of f, and thus that $f(x) \geq 0$ for all real numbers x. □