Theorem 1. *If A, B, and C are sets, then $(A \cap B) \times C = (A \times C) \cap (B \times C)$.*

Proof. We will prove that if A, B, and C are sets, then $(A \cap B) \times C = (A \times C) \cap (B \times C)$ by using element chasing to show that each set is a subset of the other.

First, let (x, y) be an element of $(A \cap B) \times C)$. So $x \in A \cap B$ and $y \in C$, i.e., $x \in A$ and $x \in B$. Since $x \in A$ and $y \in C$, $(x, y) \in A \times C$. Furthermore, since $x \in B$ and $y \in C$, (x, y) is also in $B \times C$. Since $(x, y) \in A \times C$ and $(x, y) \in B \times C$, we conclude that $(x, y) \in (A \times C) \cap (B \times C)$.

Now, suppose $(x, y) \in (A \times C) \cap (B \times C)$. Then $(x, y) \in A \times C$ and $(x, y) \in B \times C$. For this to be true, x must be a member of A and B, and y must be in C. Since $x \in A$ and $x \in B$, $x \in A \cap B$. Finally, since $x \in A \cap B$ and $y \in C$, the pair $(x, y) \in (A \cap B) \times C$.

Having shown that each product is a subset of the other, we have shown that if A, B, and C are sets, then $(A \cap B) \times C = (A \times C) \cap (B \times C)$. □