Let $\sim$ be the relation on the integers whereby $a \sim b$ if and only if $a - b = 3k$ for some integer k (i.e., $a - b$ is divisible by 3).

Theorem 1. *Relation $\sim$ as defined above is an equivalence relation.*

Proof. We will prove that $\sim$ is an equivalence relation by showing that it is reflexive, symmetric, and transitive.

To show that $\sim$ is reflexive, we need to show that $a \sim a$ for all integers a. So let a be an integer. Then $a - a = 0 = 3 \times 0$ so $a \sim a$. Thus $\sim$ is reflexive.

To show that $\sim$ is symmetric, suppose a and b are integers and $a \sim b$, i.e., $a - b = 3k$ for some integer k. Then $b - a = -3k = 3(-k)$. Since k is an integer, so is $-k$, and thus $b - a$ is divisible by 3 and so $b \sim a$. We have now shown that $\sim$ is symmetric.

To show that $\sim$ is transitive, let a, b, and c be integers such that $a \sim b$ and $b \sim c$. Thus $a - b = 3k$ for some integer k and $b - c = 3n$ for some integer n. Now

$$\begin{aligned} (a - b) + (b - c) &= 3k + 3n \\ a - c &= 3(k + n) \end{aligned}$$

Since integers are closed under addition, we have shown that $a - c$ is divisible by 3, and so $a \sim c$. Therefore $\sim$ is transitive.

We have now shown that $\sim$ is reflexive, symmetric, and transitive, and therefore that $\sim$ as defined above is an equivalence relation. □