

Here is a simple example of a proof about subsets:

**Claim 1.** *The set of integer multiples of 6 between 0 and 18 is a subset of the set of integer multiples of 3 between 0 and 18.*

*Proof.* We use the definition of subset to show that the set of integer multiples of 6 between 0 and 18 is a subset of the set of integer multiples of 3 between 0 and 18. Letting  $A$  be the integer multiples of 6 between 0 and 18, i.e.,  $A = \{0, 6, 12, 18\}$ , and  $B$  be the integer multiples of 3 between 0 and 18, i.e.,  $B = \{0, 3, 6, 9, 12, 15, 18\}$ , we see by inspection that every element of  $A$  is also an element of  $B$ . Therefore  $A \subseteq B$  and we have proved that the set of integer multiples of 6 between 0 and 18 is a subset of the set of integer multiples of 3 between 0 and 18.  $\square$

And here is a more complicated proof in which you can't just check by inspection:

**Claim 2.** *The set of all integer multiples of 6 is a subset of the set of all integer multiples of 3.*

*Proof.* We use the definition of subset to show that the set of integer multiples of 6 is a subset of the set of integer multiples of 3. Suppose  $a$  is an integer multiple of 6, i.e.,  $a = 6n$  for some integer  $n$ . Noticing that  $6n = 3(2n)$ , and that  $2n$  is an integer by closure under multiplication, we see that  $3(2n)$  is an integer multiple of 3. We have thus shown that any integer multiple of 6 is also an integer multiple of 3, and so the set of integer multiples of 6 is a subset of the set of integer multiples of 3.  $\square$