Math 239 01
Prof. Doug Baldwin

Problem Set 3 — Propositional Logic

Complete by **Monday, February 18**
Grade by **Wednesday, February 20**

Purpose

This problem set mainly develops your ability to work with propositional logic, including equivalence proofs for logical statements. It also gives you some practice working with sets and predicates. When you finish this problem set you should be able to ...

- Prove equivalence of logical statements by using truth tables
- Prove equivalence of logical statements by using Boolean algebra
- Write roster method descriptions of sets
- Find truth sets of predicates
- Write formal proofs

Background

This problem set is based on sections 2.2 and 2.3 of our textbook. We discussed this material in class between February 6 and February 11.

Activity

Answer the following questions. All proofs must be printed according to conventions for formal proofs, including typeface rules (e.g., italic variables, emphasized labels for theorems and proofs, etc.)

Question 1. Use Boolean algebra (i.e., other known equivalencies) to prove that if P and Q are propositions, then $P \to Q \equiv \neg Q \to \neg P$.

Question 2. Exercise 5a from section 2.2 of our textbook (use truth tables to prove that "or" distributes over "and").

Question 3. Exercise 9c from section 2.2 of our textbook (use previously proven logical equivalencies to prove that if P and Q are propositions, then $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$).

Question 4. Use the roster method to describe each of the following sets:

1. The set of even natural numbers greater than 1 but less than 1001.
2. The set of real numbers x such that $2x + 3 = 11$.
3. The set of positive integer multiples of 4.

4. The set of integers that can be computed by multiplying 7 by a positive even integer.
5. The set of rational numbers x such that $x^2 + 2 = 0$.

Question 5. Use the roster method to write the truth set of each of the following predicates, assuming the given universal sets U:

1. $P(x)$ is "$-4 < x < 4$"; $U = \mathbb{Z}$
2. $P(y)$ is "$3y^2 = 12$"; $U = \mathbb{N}$
3. $P(c)$ is "c is a color found on the standard US flag"; $U = \{\text{yellow}, \text{blue}, \text{green}, \text{red}\}$
4. $Q(x)$ is "$\sin x = 1$"; $U = \mathbb{Z}$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.