Math 239 01
Prof. Doug Baldwin

Last Problem Set — Infinite Sets

Complete by **Thursday, May 2**
Grade by **Monday, May 6**

Purpose

This problem set mainly develops your ability to reason about infinite sets. It also gives you some practice working with equivalence relations. When you finish this problem set you should be able to ...

- Prove that relations are or are not equivalence relations
- Recognize when sets are finite, countably infinite, and uncountable
- Prove that a set is infinite
- Prove that a set is countable
- Write formal proofs.

Background

Finite and infinite sets are discussed in chapter 9 of our textbook. We will address that material in class from April 26 through the end of the semester.

Equivalence relations and equivalence classes are discussed in sections 7.2 and 7.3 of the textbook. We covered that material in classes on April 22 and 24.

Activity

Solve the following problems. All proofs must be written according to conventions for formal proofs, including typeface rules (e.g., italic variables, emphasized labels for theorems and proofs, etc.).

Question 1. A variation on exercise 10a in section 7.2 of our textbook: define relation $\sim$ on $\mathbb{Z}$ by $a \sim b$ if and only if 2 divides $a + b$. Prove that $\sim$ is an equivalence relation, and then describe the equivalence class of the integer 1 for relation $\sim$.

Question 2. Define a "nearly equal" relation on real numbers by x is nearly equal to y if and only if $|x - y| < 0.001$. Is this "nearly equal" an equivalence relation? Explain why or why not.

(This relation comes from a real-world problem, namely that computers can't do arithmetic on real numbers exactly. This stems from the fact that computers represent real numbers in a kind of scientific notation with a fixed number of digits. Any real number that can't be exactly represented in that many digits therefore has to be rounded. One consequence is that when programmers are comparing the results of real-valued calculations (e.g., testing to see if two different functions produce the same result) they don't want to test for exact equality, because two results that are mathematically equal might be unequal in the computer due to different rounding in intermediate calculations. Thus programmers often want a "nearly equal" test for numbers. But because "nearly equal" tests are supposed to behave like equality tests, except allowing for rounding errors, it is desirable for them to be equivalence relations.)

Question 3. For each of the following sets, say whether the set is finite, countably infinite, or uncountable. You should be able to justify your choices, but do not have to give formal proofs.

1. The set of perfect squares, $\{n^2 | n \in \mathbb{Z}\}$.
2. The set of integers between -3 and 2, $\{n \in \mathbb{Z} | -3 \leq n \leq 2\}$
3. The set of real numbers between -3 and 2, $\{x \in \mathbb{R} | -3 \leq x \leq 2\}$

Question 4. Exercise 5c in section 9.1 of our textbook: prove that if A and B are sets and $A \cap B$ is infinite, then A is infinite.

Question 5. Exercise 2b in section 9.2 of our textbook: prove that the set of integers that are multiples of 5 is countably infinite.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above.