Theorem 1. If P and Q are propositions, then $\neg(P \leftrightarrow Q) \equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$.

Proof. We assume P and Q are propositions, and show via truth tables that $\neg(P \leftrightarrow Q) \equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$. The truth table for $\neg(P \leftrightarrow Q)$ is ...

P	Q	$P \leftrightarrow Q$	$\neg(P \leftrightarrow Q)$
T	T	T	F
T	F	F	T
F	T	F	T
F	F	T	F

Similarly, the truth table for $(P \vee Q) \wedge (\neg P \vee \neg Q)$ is ...

P	Q	$\neg P$	$\neg Q$	$P \vee Q$	$\neg P \vee \neg Q)$	$(P \vee Q) \wedge (\neg P \vee \neg Q)$
T	T	F	F	T	F	F
T	F	F	T	T	T	T
F	T	T	F	T	T	T
F	F	T	T	F	T	F

Since the final columns of both tables are the same, we have proven that if P and Q are propositions, then $\neg(P \leftrightarrow Q) \equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$. □

Theorem 2. If P, Q, and R are propositions, then $\neg(P \rightarrow (Q \vee R)) \equiv P \wedge \neg Q \wedge \neg R$.

Proof. We assume that P, Q, and R are propositions, and show via Boolean algebra that $\neg(P \rightarrow (Q \vee R)) \equiv P \wedge \neg Q \wedge \neg R$. Specifically, from the negation of a conditional and one of De Morgan's laws we see ...

$$\begin{aligned} \neg(P \rightarrow (Q \vee R)) &\equiv P \wedge \neg(Q \vee R) \\ &\equiv P \wedge \neg Q \wedge \neg R \end{aligned}$$

We have thus shown that if P, Q, and R are propositions, then $\neg(P \rightarrow (Q \vee R)) \equiv P \wedge \neg Q \wedge \neg R$. □