Conjecture 1. If n is an odd integer, then $n \equiv 1 \pmod 2$.

Proof. We assume n is an odd integer, and will show that $n \equiv 1 \pmod 2$. In other words, from the definition of congruence, we will show that $1 - n$ is an integer multiple of 2. Since n is odd, we have $n = 2a + 1$ for some integer a, and so

$$\begin{aligned} 1 - n &= 1 - (2a + 1) \\ &= -2a \\ &= 2(-a) \end{aligned}$$

Since integers are closed under multiplication (by negative 1 in this case), $-a$ is an integer, and so $1 - n$ is an integer multiple of 2. We have therefore shown that if n is an odd integer, then $n \equiv 1 \pmod 2$. □