1 Quotation Marks

Use pairs of backward and forward single quote keys to create opening and closing “quotation marks” in LaTeX.

2 Proofs via the Contrapositive

Theorem 1. For all integers n, if n is not divisible by 2, then n is also not divisible by 4.

Proof. We assume that n is an integer, and will show by using the contrapositive that if n is not divisible by 2, then n is also not divisible by 4. The contrapositive of this statement is if n is divisible by 4, then n is divisible by 2. Since n is divisible by 4, $n = 4a$ for some integer a. Then

$$\begin{aligned} n &= 4a \\ &= 2(2a) \end{aligned}$$

Since integers are closed under multiplication, $2a$ is an integer, and so we have shown that n is divisible by 2. Thus by proving the contrapositive, we have proven that if n is not divisible by 2, then n is also not divisible by 4. □

Theorem 2. For all integers n not equal to 0, if x is irrational, then nx is also irrational.

Proof. We assume that n is an integer and not equal to 0, and show via the contrapositive that if x is irrational, then nx is also irrational. The contrapositive of this statement is if nx is rational then x is rational. So assume nx is rational, i.e.,

$$nx = \frac{a}{b} \tag{1}$$

for some integers a and b with $b \neq 0$. Now, since n is not equal to 0, we can divide both sides of [**??**] by n to get

$$x = \frac{a}{nb}$$

Since integers are closed under multiplication, nb is an integer, and since neither n nor b are 0, $nb \neq 0$. Thus we see that x is rational. This proves the contrapositive of the theorem statement, and so we have shown that for all integers n not equal to 0, if x is irrational, then nx is also irrational. □