**Theorem 1.** The set $F$ of all functions from $\mathbb{N}$ to $\mathbb{N}$ is uncountable.

*Proof.* We prove by contradiction that the set $F$ of all functions from $\mathbb{N}$ to $\mathbb{N}$ is uncountable. Assume for the sake of contradiction that $F$ is countable. Then there is a bijection $f : \mathbb{N} \to F$ that allows us to number the members of $F$ as $f_1$, $f_2$, etc. Now construct a new function $g : \mathbb{N} \to \mathbb{N}$ by $g(n) = f_n(n) + 1$. Clearly $g$ is a member of $F$, yet it cannot be because it differs from each element of $F$ in how it maps at least one number. It is impossible for $g$ to simultaneously be a member of $F$ and not be, and so we conclude that the set $F$ of all functions from $\mathbb{N}$ to $\mathbb{N}$ is in fact not countable. $\square$