Theorem 1. Every natural number $n \geq 6$ can be written as $3a + 4b$ for some non-negative integers a and b.

Proof. We show by strong induction on n that every natural number $n \geq 6$ can be written as $3a + 4b$ for some non-negative integers a and b.

For the basis steps we show that 6, 7, and 8 can be written as $3a + 4b$ for non-negative integers a and b. In particular ...

- $6 = 3 \times 2 + 4 \times 0$
- $7 = 3 \times 1 + 4 \times 1$
- $8 = 3 \times 0 + 4 \times 2$

For the induction step, we assume that for some $k \geq 8$, all natural numbers, i, such that $6 \leq i \leq k$ can be written as $3a + 4b$. We can then write $k + 1$ as $(k - 2) + 3$. Since $k \geq 8$, we know that $k - 2 \geq 6$ and therefore can be written as $3a + 4b$ for some non-negative integers a and b. Thus $k + 1 = (3a + 4b) + 3 = 3(a + 1) + 4b$.

Since we have shown that 6, 7, and 8 can be written as $3a + 4b$ for non-negative integers a and b, and that for any $k \geq 8$, if all naturals between 6 and k can be written as $3a + 4b$ then so can $k + 1$, we have proven by strong induction that every natural number $n \geq 6$ can be written as $3a + 4b$ for some non-negative integers a and b. □