Math 239 — Hour Exam 1

February 16, 2018

General Directions. This is an open-book, open-notes, open-computer test. However, you may not communicate with any person, except me, during the test. You have the full class period (50 minutes) in which to do the test. Put your answer to each question in the space provided (use the backs of pages if you need more space). Be sure to **show your work**! I give partial credit for incorrect answers if you show correct steps leading up to them; conversely, I do not give full credit even for correct answers if it is not clear that you understand where those answers come from. Good luck.

This test contains 5 questions on 4 pages.

Question 1 (5 Points). Using the roster method, describe the set that contains all (and only) the numbers of the form $200n - 3$ where n can be any natural number. For example, this set contains 197, 397, 597, and all similar numbers.

Solution. Since n is a natural number, the smallest element of the set is $200 \times 1 - 3 = 197$. Adding 200 gives the next element, and doing this a few times produces enough elements to show a pattern:

$$\{197, 397, 597, 797, \cdots\}$$

Question 2 (10 Points). Use set builder notation to describe the set that contains all (and only) the numbers that are integer powers of 3/2. For example, this set contains $(3/2)^0$, $(3/2)^1$, $(3/2)^2$, $(3/2)^{-1}$, $(3/2)^{-2}$, and all similar numbers.

Solution. From the problem description, the set contains every number produced by raising 3/2 to an integer power. This suggests the following "calculational" set builder form:

$$\left\{\left(\frac{3}{2}\right)^n \mid n \in \mathbb{Z}\right\}$$

Question 3 (5 Points). Is the exclamation "Hooray!" a mathematical statement? Why or why not?

Solution. "Hooray" is not a mathematical statement because it is neither true nor false.

Question 4 (20 Points). Write a formal proof of the following theorem:

Theorem: If n is an odd integer, then $-n$ is also an odd integer.

Your proof should follow all relevant conventions of formal proof writing, except that it may be written by hand instead of typeset, and thus needn't follow conventions that require specific typefaces (e.g., italicizing variable names).

Solution.

Proof. We assume that n is an odd integer, and will show that $-n$ is also an odd integer. Since n is odd, it can be written as

$$n = 2a + 1$$

for some integer a. Negating both sides of this equation gives

$$\begin{aligned} -n &= -(2a + 1) \\ &= -2a - 1 \\ &= -2a - 2 + 1 \\ &= 2(-a - 1) + 1 \end{aligned}$$

Since the integers are closed under multiplication by -1 and subtraction, $-a-1$ is an integer, and thus $-n$ has been shown to be of the form $2b+1$ for some integer b, i.e., to be an odd integer. We have thus shown that if n is an odd integer, then $-n$ is also an odd integer. QED.

Question 5 (10 Points). Show (not necessarily in a formal proof) that the logical expression $(P \vee Q) \wedge \neg(P \wedge Q)$ is equivalent to $(P \wedge \neg Q) \vee (\neg P \wedge Q)$.

Solution. You can show this through Boolean algebra:

$$(P \vee Q) \wedge \neg(P \wedge Q) \equiv (P \vee Q) \wedge (\neg P \vee \neg Q)$$

$$\equiv \big(P \wedge (\neg P \vee \neg Q)\big) \vee \big(Q \wedge (\neg P \vee \neg Q)\big)$$

$$\equiv (P \wedge \neg P) \vee (P \wedge \neg Q) \vee (Q \wedge \neg P) \vee (Q \wedge \neg Q)$$

$$\equiv (P \wedge \neg Q) \vee (\neg P \wedge Q)$$

(You could also use a truth table, although I haven't shown that approach here.)