Theorem 1. Let $U = \{1, 2, 3, \ldots, 99\}$ be a universal set, and for each integer i such that $1 \leq i \leq 9$, let A_i be the set $A_i = \{n \in U | \text{the first digit of the base-10 representation of } n\text{, without leading zeros, is } i\}$. Then the family $\mathcal{A} = \{A_1, A_2, A_3, \ldots, A_9\}$ is a partition of U.

Proof. We prove that $\mathcal{A}$ is a partition of U by showing that (1) every element of U is in some A_i, (2) no element of U is in more than one A_i, and (3) no A_i is empty.

To show that every element of U is in some A_i, note that every number in U has a base-10 representation, and that those representations must start with one of the digits 1 through 9. Therefore each number is in one of A_1 through A_9.

We show that no element of U is in more than one A_i by contradiction. Suppose n is in A_i and A_j where $i \neq j$. Then the base-10 representation of n begins with i and with j, which is impossible because every integer has a unique base-10 representation.

To show that no A_i is empty, notice that for each integer $i, 1 \leq i \leq 9$, i is in A_i.

Having shown each of the requirements for being a partition, we have proven that $\mathcal{A}$ is a partition of $U = \{1, 2, 3, \ldots, 99\}$. $\square$