Theorem 1. The set of irrational numbers is uncountable.

Proof. We prove by contradiction that the set of irrational numbers is uncountable. Assume for the sake of contradiction that the set of irrational numbers is countable. Now we know that the set of real numbers is the union of the set of irrationals and the set of rationals, and that the set of rationals is countable. Furthermore, the union of two countable sets is countable, and so under our assumption, the set of real numbers must be countable. But this contradicts the fact that the set of reals is in fact uncountable. Having reached a contradiction from the assumption that the set of irrationals is countable, we conclude that in fact the set of irrational numbers is uncountable. □