Theorem 1. The set $S = \{n \in \mathbb{Z} | n > -30\}$ is inductive.

Proof. To prove that S is inductive, we need to show that S is a subset of the integers and that if $n \in S$, then $n+1 \in S$.

We first show that if $n \in S$, then $n+1 \in S$. This follows from the fact that if $n \in S$ then $n > -30$ and so $n+1 > -29 > -30$.

To show that S is a subset of $\mathbb{Z}$, we notice from the definition of S that all elements of S must be integers, and so $S \subseteq \mathbb{Z}$.

Having shown that if $n \in S$, then $n+1 \in S$ and that $S \subseteq \mathbb{Z}$, we have proven that S is inductive. □

Theorem 2. If the universal set is the natural numbers, then the truth set of the predicate $P(n) =$ "4 divides $(5^n - 1)$" is inductive.

Proof. We assume the universal set is the natural numbers, and will show that the truth set of the predicate $P(n) =$ "4 divides $(5^n - 1)$" is inductive. Call the truth set T. To prove that T is inductive, we need to show that it is a subset of the integers and that if $n \in T$, then $n+1 \in T$.

To show that T is a subset of the integers, note that T is a subset of the universal set, which is a subset of the integers.

Next, we assume that n is in T, i.e., $5^n - 1 = 4a$ for some integer a, and we show that $n+1$ is in T, i.e., $5^{n+1} - 1 = 4b$ for some integer b. We can write $5^{n+1} - 1$ as...

$$\begin{aligned} 5^{n+1} - 1 &= 5 \times 5^n - 5 + 4 \\ &= 5(5^n - 1) + 4 \\ &= 5(4a) + 4 \\ &= 4(5a + 1) \end{aligned}$$

Since the integers are closed under multiplication and addition, $5a + 1$ is an integer, and so 4 divides $5^{n+1} - 1$. Thus whenever $n \in T$, $n+1 \in T$.

Having shown that T is a subset of the integers, and that if $n \in T$, $n + 1 \in T$, we have shown that T is inductive. We have thus proven that if the universal set is the natural numbers, then the truth set of the predicate $P(n) =$ "4 divides $(5^n - 1)$" is inductive. □