Let $\sim$ be a relation on the real numbers, defined as $x \sim y$ if and only if $x^2 = y^2$.

Theorem 1. Relation $\sim$ is an equivalence relation.

Proof. We show that $\sim$ is an equivalence relation by showing that it is reflexive, symmetric, and transitive.

To show that $\sim$ is reflexive, note that for all real numbers x, $x^2 = x^2$, so $x \sim x$.

To show that $\sim$ is symmetric, let x and y be real numbers such that $x \sim y$, i.e., $x^2 = y^2$. Therefore $y^2 = x^2$,and so $y \sim x$.

To show that $\sim$ is transitive, we assume that $x \sim y$ and $y \sim z$, and show that $x \sim z$. In other words $x^2 = y^2$ and $y^2 = z^2$. Adding these equations, we get

$$x^2 + y^2 = y^2 + z^2$$

Subtracting y^2 from both sides yields

$$x^2 = z^2$$

and thus $x \sim z$.

Since we have shown that $\sim$ is reflexive, symmetric, and transitive, we have proven that $\sim$ is an equivalence relation. □

Or we can use a shorter argument that $\sim$ is transitive:

Proof. We show that $\sim$ is an equivalence relation by showing that it is reflexive, symmetric, and transitive.

To show that $\sim$ is reflexive, note that for all real numbers x, $x^2 = x^2$, so $x \sim x$.

To show that $\sim$ is symmetric, let x and y be real numbers such that $x \sim y$, i.e., $x^2 = y^2$. Therefore $y^2 = x^2$,and so $y \sim x$.

To show that $\sim$ is transitive, we assume that $x \sim y$ and $y \sim z$, and show that $x \sim z$. In other words $x^2 = y^2$ and $y^2 = z^2$. Since equality is transitive, $x^2 = y^2$ and $y^2 = z^2$ implies that $x^2 = z^2$, and thus $x \sim z$.

Since we have shown that $\sim$ is reflexive, symmetric, and transitive, we have proven that $\sim$ is an equivalence relation. □