Theorem 1. The set $S = \{2^i | i \in \mathbb{N}\}$ is countably infinite.

Proof. We prove that S is countably infinite by showing that there is a bijection $f : \mathbb{N} \to S$. Consider $f(n) = 2^n$; it is certainly a function from $\mathbb{N}$ to S. Function f is an injection because if $f(n) = f(m)$ then $2^n = 2^m$ and so $n = m$. Furthermore, f is a surjection because if s is any element of S, then $s = 2^n$ for some natural number n, and so $s = f(n)$. Since we have shown that f is an injection and a surjection from $\mathbb{N}$ to S, we have shown that there is a bijection from $\mathbb{N}$ to S and therefore S is countably infinite. $\square$