Theorem 1. If A and B are subsets of some universal set, then $A - B \subseteq A \cap B^C$.

Proof. We assume that A and B are subsets of some universal set, and show that $A - B \subseteq A \cap B^C$. Let x be an element of $A - B$. Therefore $x \in A$ but $x \notin B$. Since $x \notin B$, x is in B^C. Finally, since $x \in A$ and $x \in B^C$, $x \in A \cap B^C$. We have thus shown that if A and B are subsets of some universal set, then $A - B \subseteq A \cap B^C$. $\square$

Theorem 2. If $A = \{x \in \mathbb{Z} | x \equiv 1 \pmod 4\}$ and $B = \{y \in \mathbb{Z} | y \equiv 2 \pmod{10}\}$, then A and B are disjoint.

Proof. We assume $A = \{x \in \mathbb{Z} | x \equiv 1 \pmod 4\}$ and $B = \{y \in \mathbb{Z} | y \equiv 2 \pmod{10}\}$, and show that A and B are disjoint. The proof is by contradiction. Assume for the sake of contradiction that A and B are not disjoint, i.e., there is some integer, call it z, in both sets. In order to be in set A, z must equal $4a + 1$ for some integer a, and in order to be in set B z must equal $10b + 2$ for some integer b. We therefore see that

$$\begin{aligned} 4a + 1 &= 10b + 2 \\ 4a &= 10b + 1 \\ 2(2a) &= 2(5b) + 1 \end{aligned}$$

The left side of this last equation describes an even integer, but the right side describes an odd integer. Since it is impossible for an even integer to also be odd, we have arrived at a contradiction. We have thus proven by contradiction that if $A = \{x \in \mathbb{Z} | x \equiv 1 \pmod 4\}$ and $B = \{y \in \mathbb{Z} | y \equiv 2 \pmod{10}\}$, then A and B are disjoint. $\square$