Define $f(x)$ as follows:

$$f(x) = \begin{cases} x^2 & \text{if } x < 0, \\ 2x & \text{if } x \geq 0 \end{cases} \tag{1}$$

Then we have ...

Theorem 1. *If $f(x)$ is defined by Equation 1, then $f(x)$ is non-negative for all real numbers x.*

Proof. We assume that $f(x)$ is defined by Equation 1, and will show that $f(x)$ is non-negative for all real numbers x. We do the proof by cases, based on the cases in the definition of f.

For the first case, suppose $x < 0$. Then $f(x) = x^2$, and since the square of a negative number is positive, $f(x)$ is non-negative.

For the second case, suppose $x \geq 0$, i.e., x is non-negative. Then $f(x) = 2x$. Since twice a non-negative number is non-negative, $f(x)$ is non-negative.

Since we have now shown that $f(x)$ is non-negative in all the cases in Equation 1, we have shown that if $f(x)$ is defined by Equation 1, then $f(x)$ is non-negative for all real numbers x. □

Theorem 2. *Every integer multiple of 5 is either of form $10n$ or $10n + 5$, for some integer n.*

Proof. We assume that x is an integer multiple of 5, and will show that either $x = 10n$ or $x = 10n + 5$ for some integer n. Since x is a multiple of 5, we can write x as $x = 5k$ for some integer k. The proof then proceeds by cases, depending on whether k is odd or even.

For the first case, suppose k is odd. In other words, $k = 2n + 1$ for some integer n. Then

$$\begin{aligned} x &= 5k \\ &= 5(2n+1) \\ &= 10n + 5 \end{aligned}$$

We thus see that x is of the form $x = 10n + 5$.

For the second case, suppose k is even. In other words, $k = 2n$ for some integer n. Then

$$\begin{aligned} x &= 5k \\ &= 5(2n) \\ &= 10n \end{aligned}$$

We thus see that x is of the form $x = 10n$.

Since all integers are either odd or even, we have shown that every integer multiple of 5 is of the form $10n$ or $10n + 5$ for some integer n. □