Math 239 Problem Set 2 Solution

January 27, 2017

Here are my proofs for the three propositions in Problem Set 2.

Proposition 1. If x is an even integer and y is an even integer, then $x + y$ is an even integer.

Proof. We let x and y be even integers, and show via a direct proof that $x + y$ is also even. Since x and y are even, there must be integers a and b such that $x = 2a$ and $y = 2b$. Substituting these expressions into $x + y$ yields

$$\begin{aligned} x + y &= 2a + 2b \\ &= 2(a + b) \\ &= 2c \end{aligned}$$

Where c is an integer because the integers are closed under addition. Since $x + y = 2c$ for some integer c, we conclude that $x + y$ is even. $\square$

Proposition 2. If m is an odd integer, then $5m + 7$ is an even integer.

Proof. We let m be an odd integer, and show via a direct proof that $5m+7$ is an even integer. From Sundstrom's Theorem 1.8 (the product of two odd integers is an odd integer), $5m$ must be an odd integer. In other words, there must be some integer a such that $5m = 2a+1$. Also note that $7 = 2 \times 3+1$. Using these equalities, we can express $5m + 7$ as

$$\begin{aligned} 5m + 7 &= 2a + 1 + 2 \times 3 + 1 \\ &= 2(a + 3) + 2 \\ &= 2(a + 4) \\ &= 2b \end{aligned}$$

Where b is an integer because the integers are closed under addition. Since $5m + 7 = 2b$ for some integer b, we conclude that $5m + 7$ is even. $\square$

Proposition 3. If x_1 and x_2 are the solutions to the quadratic equation $ax^2 + bx + c = 0$, then $x_1 + x_2 = -\frac{b}{a}$.

Proof. We let x_1 and x_2 be the solutions to the quadratic equation $ax^2+bx+c = 0$, and show via a direct proof that

$$x_1 + x_2 = -\frac{b}{a}$$

Using the quadratic formula, we see without loss of generality that

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \tag{1}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \tag{2}$$

Adding (1) and (2) gives

$$\begin{aligned} x_1 + x_2 &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2b}{2a} \end{aligned}$$

Simplifying the last of these equations establishes that $x_1 + x_2 = -\frac{b}{a}$. □