Math 239 Exam 1 Solution

February 15, 2017

Here are my solutions to the questions on hour exam 1.

Problem 1. (Use set builder notation to define the set of third powers of even natural numbers.)

Even natural numbers are numbers of the form $2n$ for some integer $n > 0$. Having $n > 0$ conveniently means that n is a natural number, not just any integer. Third powers of even naturals are numbers of the form $(2n)^3$. Putting these ideas together and using the second form of set builder notation gives

$$\{(2n)^3 | n \in \mathbb{N}\}$$

Problem 2. (State the conjecture that the set of integer multiples of 13 is closed under addition as a theorem, and give a formal proof of it.)

Theorem 1. If x and y are members of the set $M = \{13k | k \in \mathbb{Z}\}$ then $x + y$ is a member of M.

Proof. Assume that x and y are members of M. Then from the definition of M, $x = 13a$ for some integer a and $y = 13b$ for some integer b. We can then express $x + y$ as

$$\begin{aligned} x + y &= 13a + 13b \\ &= 13(a + b) \\ &= 13c \end{aligned}$$

where c is an integer since the integers are closed under addition. Since $x + y = 13c$ for some integer c, we have shown that if x and y are members of M then $x + y$ is a member of M. □

Problem 3. (Show that $P \wedge P$ is equivalent to $P \vee P$.)

Construct a truth table that compares $P \wedge P$ to $P \vee P$:

P	$P \wedge P$	$P \vee P$
T	T	T
F	F	F

Since the columns for $P \wedge P$ and $P \vee P$ are identical, the two expressions are equivalent.

Problem 4. (Given that $P(x)$ is "if x is odd then $x^2 < 0$" and S is the set $\{n \in \mathbb{Z} | P(n)\}$, determine whether S is empty or not.)

S is the set of all integers for which P is true. P is a conditional statement. There are no integers for which its conclusion, $x^2 < 0$, is true, but as a conditional P is also true whenever its hypothesis is false, which it is for all even integers. So S is the set of even integers, and so, for example, 2 is a member.

Problem 5. (What follows from both "if Judy is a millionaire then I'm the king of Spain" and "I am not the king of Spain" being true?)

Since the conclusion to the conditional is false, the only way for the conditional as a whole to be true is for its hypothesis to also be false. Thus we conclude that Judy is not a millionaire.