Theorem 1. For all even integers, x, and odd integers, y, $x + y$ is odd.

Proof. We assume that x and y are even and odd integers respectively, and will show that $x + y$ is odd. Using the definitions of even and odd integers, we see that

$$x = 2a \tag{1}$$

for some integer a, and

$$y = 2b + 1 \tag{2}$$

for some integer b. Expressing $x + y$ in terms of 1 and 2, we get

$$
\begin{aligned}
x + y &= 2a + 2b + 1 \\
&= 2(a + b) + 1 \\
&= 2c + 1
\end{aligned}
$$

for some integer c, since integers are closed under addition. Since $x + y = 2c + 1$ for some integer c, we conclude that $x + y$ is odd. $\square$