Math 239 Problem Set 3 Solution

Problem 1. Construct a truth table for $P \leftrightarrow Q$

P	Q	$Q \to P$	$P \to Q$	$Q \to P \wedge P \to Q$
T	T	T	T	T
T	F	T	F	F
F	T	F	T	F
F	F	T	T	T

Problem 2. Show that $(P \to Q) \wedge P) \to Q)$ is a tautology.

The easiest way to show this is via a truth table:

P	Q	$P \to Q$	$(P \to Q) \wedge P$	$((P \to Q) \wedge P) \to Q$
T	T	T	T	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

Since the last column contains only "True," the statement is indeed a tautology.

Problem 3. Give a negation of "if you graduate from college, then you will get a job or go to graduate school."

Since the negation of $P \to Q$ is $P \wedge \neg Q$, this negation could be "you graduate from college but do not get a job or go to graduate school," keeping in mind that the "not" in the conclusion applies to "get a job or go to graduate school." This could be made a bit clearer, thanks to one of De Morgan's laws, as "you graduate from college but do not get a job and do not go to graduate school."

Problem 4. Give a formal proof, using truth tables, that if P, Q, and R are propositions, then $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$.

Proof. We assume that P, Q, and R are propositions, and show that $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$. Using a truth table with the two sides of the equivalence in its last columns, we see

P	Q	R	$Q \wedge R$	$P \vee Q$	$P \vee R$	$P \vee (Q \wedge R)$	$(P \vee Q) \wedge (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	F	T	T	T	T
T	F	T	F	T	T	T	T
T	F	F	F	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	F	T	F	F	F
F	F	T	F	F	T	F	F
F	F	F	F	F	F	F	F

Since the rightmost two columns of the truth table are identical, we have proven that if P, Q, and R are propositions, then $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$. $\square$

Problem 5. Give a formal proof, using equivalencies, that if P and Q are propositions, then $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$.

Proof. We assume that P and Q are propositions, and show that $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$. Using the definition of the biconditional and one of De Morgan's law, we see

$$\begin{aligned} \neg(P \leftrightarrow Q) &\equiv \neg((P \to Q) \wedge (Q \to P)) \\ &\equiv \neg(P \to Q) \vee \neg(Q \to P) \end{aligned}$$

Now using the equivalence $A \to B \equiv \neg A \vee B$ and one of De Morgan's laws we have

$$\begin{aligned} \neg(P \to Q) \vee \neg(Q \to P) &\equiv \neg(\neg P \vee Q) \vee \neg(\neg Q \vee P) \\ &\equiv (P \wedge \neg Q) \vee (Q \wedge \neg P) \end{aligned}$$

We have thus shown that if P and Q are propositions, then $\neg(P \leftrightarrow Q) \equiv (P \wedge \neg Q) \vee (Q \wedge \neg P)$. $\square$