Math 239 Problem Set 2 Solution

Proposition 1 (Section 1.2 exercise 2a)**.** If x and y are even integers, then $x + y$ is an even integer.

Proof. We assume that x and y are even integers, and show that $x + y$ is an even integer. Since x is an even integer, there is some other integer m such that $x = 2m$. Since y is also an even integer, there is another integer n such that $y = 2n$. We now calculate $x + y$ as

$$\begin{aligned} x + y &= 2m + 2n \\ &= 2(m + n) \end{aligned}$$

Letting $k = m + n$, we have $x + y = 2k$. Since the integers are closed under addition, k is an integer and so $x + y$ is even by the definition of even integer. We have thus proven that if x and y are even integers, then $x + y$ is an even integer. □

Proposition 2 (Section 1.2, exercise 4b)**.** If m is an odd integer, then $5m + 7$ is an even integer.

Proof. We assume that m is an odd integer, and show that $5m + 7$ is an even integer. From Theorem 1.8 (if x and y are odd integers, then xy is an odd integer) and the fact that 5 and m are both odd integers, $5m$ must be an odd integer. From the proposition in exercise 2c of section 1.2 (if x and y are odd integers, then $x + y$ is an even integer, the proof appearing on page 537)[1], we further see that $5m + 7$ must be even. Thus we have proven that if m is an odd integer, then $5m + 7$ is an even integer. □

Proposition 3 (Section 1.2, exercise 11a)**.** If a, b, and c are real numbers and $a \neq 0$, then the sum of the solutions to the quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{a}$.

Proof. We assume that a, b, and c are real numbers and $a \neq 0$, and show that the sum of the solutions to the quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{a}$. Let

[1] Here I am using the idea that any proposition stated before an exercise, and whose proof has been given in class, developed through an earlier exercise, or appears in our textbook, may be used in that exercise

x_1 and x_2 be the solutions to the equation; specifically let

$$x_1 = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$$

and

$$x_2 = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

Then using algebra we have

$$\begin{aligned} x_1 + x_2 &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-2b}{2a} \\ &= \frac{-b}{a} \end{aligned}$$

We have thus proven that if a, b, and c are real numbers and $a \neq 0$, then the sum of the solutions to the quadratic equation $ax^2 + bx + c = 0$ is $\frac{-b}{a}$. $\square$