

Example Proofs in Propositional Logic

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Theorem 1. If P , Q , and R are propositions, then $P \rightarrow (Q \vee R) \equiv (P \wedge \neg Q) \rightarrow R$

Proof. We let P , Q , and R be propositions, and show that $P \rightarrow (Q \vee R) \equiv (P \wedge \neg Q) \rightarrow R$. Using the equivalence $A \rightarrow B \equiv \neg A \vee B$ several times and Boolean algebra we see

$$\begin{aligned} P \rightarrow (Q \vee R) &\equiv \neg P \vee (Q \vee R) \\ &\equiv (\neg P \vee Q) \vee R \\ &\equiv \neg(\neg P \vee Q) \rightarrow R \\ &\equiv (\neg\neg P \wedge \neg Q) \rightarrow R \\ &\equiv (P \wedge \neg Q) \rightarrow R \end{aligned}$$

Thus if P , Q , and R are propositions, then $P \rightarrow (Q \vee R) \equiv (P \wedge \neg Q) \rightarrow R$. \square