Math 223 01
Prof. Doug Baldwin

# Problem Set 8 — Arc Length and Curvature

Complete by **Wednesday, March 4**
Grade by **Friday, March 6**

## Purpose

This problem set reinforces your understanding of arc length and curvature of vector valued functions. By the time you finish this problem set, I expect that you will be able to

- Use the formula for arc length of a vector-valued function to find arc lengths and solve related problems.
- Use the formulas for curvature and related quantities to solve problems.
- Use Mathematica to assist in solving problems related to vector-valued functions.

## Background

This exercise is based on section 12.4 of our textbook, which we covered in classes between February 21 and 26.

## Activity

Solve the following problems. For questions that ask you to use Mathematica, please do not use the `ArcLength` and `ArcCurvature` functions (more exactly, I expect you to be able to solve the problems without those functions; if you do so and then use the functions to check your answers, that's a fine way to go slightly beyond what I expect).

**Question 1.** Find the length of one turn of the 4-dimensional helix $\mathbf{r}(t) = \langle 2\sin t, \sqrt{3}t, 2\cos t, 2t\rangle$.

(Be prepared when grading this problem to discuss how you came up with the formula(s) you used for the calculation and whether it even makes sense to talk about a length in 4 dimensions.)

**Question 2.** Set up an integral to find the circumference of the ellipse $\mathbf{r}(t) = \langle \cos t, 2\sin t\rangle$. Use Mathematica to numerically evaluate this integral (you won't be able to do it by hand).

**Question 3.** An ant crawls along the curve $\mathbf{r}(t) = \langle 2t^2, t^2 - 1, \frac{\sqrt{5}}{2}t^2\rangle$, starting at $\mathbf{r}(1)$. The ant moves in the direction of increasing $t$, i.e., it moves through points on the curve associated with ever larger $t$ values. Find the coordinates of the point the ant is at after it has crawled a distance of 1 unit.

**Question 4.** Find a unit vector that points in the direction the curve $\mathbf{r}(t) = \langle \cos(e^t), \sin(e^t), 0\rangle$ is turning when $t = \ln \pi$. Also find the curvature of $\mathbf{r}(t)$. Use Mathematica to carry out and organize the calculations (but remember not to use the `ArcCurvature` function).

## Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.