Math 223 01
Prof. Doug Baldwin

Problem Set 7 — Derivatives and Integrals of Vector-Valued Functions

Complete by **Monday, February 24**
Grade by **Wednesday, February 26**

Purpose

This problem set reinforces your understanding of derivatives and integrals of vector-valued functions. By the time you finish this problem set I expect you to be able to. . .

- Calculate derivatives of vector-valued functions
- Calculate antiderivatives and definite integrals of vector-valued functions
- Use Mathematica to find derivatives and integrals of vector-valued functions
- Use derivatives of vector-valued functions to solve problems
- Use integrals of vector-valued functions to solve problems.

Background

This problem set is based on section 12.2 of our textbook. We discussed deriatives of vector-valued functions in class on February 19, and antiderivatives and definite integrals on February 20. We introduced differentiation and integration with Mathematica in class on February 21.

Activity

Solve the following problems:

Question 1. Let $\mathbf{r}(t) = \langle t^2 + 3t, e^t \cos(e^t), \sqrt{t+1} \rangle$.

Part A. Find the derivative of $\mathbf{r}(t)$ with respect to t.

Part B. Find the general antiderivative of $\mathbf{r}(t)$ with respect to t.

Part C. Use Mathematica to confirm your answer to parts A and B.

Question 2. Find (by hand)

$$\int_0^1 \langle 1, t, t^2, t^3 \rangle \, dt$$

After finding the answer by hand, confirm it with Mathematica.

Question 3. An ant is crawling along a coil of wire in such a manner that t seconds after the ant starts crawling it is at position $\langle 2t, \sin t, \cos t \rangle$, in some coordinate system in which distance is measured in inches.

Part A. Find a function for the ant's velocity as a function of time. Do the necessary calculations by hand, but then check them with Mathematica.

Part B. Find a function for the ant's speed as a function of time. (Recall the relationship between "velocity" and "speed.")

Part C. Show that the ant's acceleration is always perpendicular to its velocity. Then explain why our book's Property vii of derivatives of vector-valued functions (see the main theorem in the "Properties of the Derivative" subsection of Section 12.2) predicts this fact.

Question 4. An object starts at rest at point $(1, 2, 0)$ and accelerates with acceleration $\mathbf{a}(t) = \langle 0, 1, 2 \rangle$ (with magnitude measured in meters per second per second). Find the location of the object after 2 seconds. After doing the necessary calculations by hand, verify your answer with Mathematica.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.