Math 223 01
Prof. Doug Baldwin

Problem Set 6 — Introduction to Vector-Valued Functions

Complete by **Thursday, February 20**
Grade by **Monday, February 24**

Purpose

This problem set develops basic understanding of vector-valued functions and their limits. By the time you finish this problem set I expect you to be able to…

- Calculate values of vector-valued functions
- Determine where, if at all, vector-valued functions take on given values
- Plot vector-valued functions with Mathematica
- Find limits of vector-valued functions and recognize when those limits do not exist.

Background

This problem set is based on section 12.1 of our textbook. We discussed the basic idea of a vector-valued function in class on February 14, and limits of vector-valued functions on February 17. Plotting vector-valued functions with Mathematica was covered in class on February 14.

Activity

Solve the following problems:

Question 1. Let

$$\mathbf{r}(t) = \langle t^2 - 1, \frac{2t}{t+1}, \sqrt{t} + 2 \rangle$$

Part A. Calculate $\mathbf{r}(1)$.

Part B. Does $\mathbf{r}(t)$ ever equal the zero vector, $\langle 0, 0, 0 \rangle$? If so, give the value(s) of t at which it does so; if not, show why no value of t can make $\mathbf{r}(t) = \langle 0, 0, 0 \rangle$.

Part C. Use Mathematica to plot $\mathbf{r}(t)$ over the interval $0 \leq t \leq 4$.

Question 2. Repeat Question 1 for

$$\mathbf{s}(t) = \langle \sin(\pi t), t \cos(\pi t), \ln(t+1) \rangle$$

Note: The natural logarithm function in Mathematica is "Log".

Question 3. Can vector-valued functions have more than 3 components? If not, explain why not. If so, give an example of such a function, and calculate its value for at least 2 t values.

Question 4. Define the sum of two vector valued functions to be the function that produces the sum of the vectors produced by the two original functions. More formally, if $\mathbf{f}(t)$ and $\mathbf{g}(t)$ are vector-valued functions, define $(\mathbf{f} + \mathbf{g})(t)$ to be $\mathbf{f}(t) + \mathbf{g}(t)$.

Using this definition, prove a "sum limit law" for vector valued functions, i.e., prove that for any real number c for which $\lim_{t\to c} \mathbf{f}(t)$ and $\lim_{t\to c} \mathbf{g}(t)$ exist,

$$\lim_{t\to c}(\mathbf{f} + \mathbf{g})(t) = \lim_{t\to c} \mathbf{f}(t) + \lim_{t\to c} \mathbf{g}(t)$$

Question 5. Find the following limits, or show that they do not exist. In all cases, be prepared to show your work when grading this problem set.

Part A.

$$\lim_{t\to 0}\langle t^2, \frac{t}{t^2+t}, 3t+1\rangle$$

Part B.

$$\lim_{t\to 2}\langle \frac{x^2-4}{|x-2|}, \frac{x^2-4}{x-2}, \frac{x-2}{x^2-4}\rangle$$

What about the limit of the same function as t approaches -2?

Part C.

$$\lim_{t\to\infty}\langle \frac{1}{t}\sin t, \frac{1}{t}\cos t, \frac{1}{t}\rangle$$

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.