Math 223 01
Prof. Doug Baldwin

Problem Set 5 — Lines and Planes

Complete by **Sunday, February 16**
Grade by **Tuesday, February 18**

Purpose

This problem set reinforces your understanding of equations for lines and planes in space. By the time you finish this problem set I expect you to be able to...

- Find equations for lines
- Find equations for planes
- Use line and plane equations to reason about relationships between lines and planes
- Carry out the vector calculations arising in connection with lines and planes.

Background

Lines and planes are the subject of section 11.5 in our textbook. We discussed them in class between February 7 and 13.

Activity

Solve the following problems:

Question 1. (An adaptation of exercise 2 in section 11.5E in our textbook.)

Part A. Give vector, parametric, and symmetric equations for the line containing points $P = (4, 0, 5)$ and $Q = (2, 3, 1)$.

Part B. Give one example of a point other than $(4, 0, 5)$ and $(2, 3, 1)$ that is on this line.

Question 2. (An adaptation of question 26 in section 11.5E of our textbook.)

Part A. Find a scalar equation for the plane that contains point $P = (3, 2, 2)$ and has normal vector $\mathbf{n} = \langle 2, 3, -1 \rangle$.

Part B. Find a point other than $(3, 2, 2)$ that is also in this plane.

Question 3. One elegant way of doing computer graphics is called "ray tracing," because it traces (in reverse) rays of light that reach a virtual eye from some virtual scene. In other words, it traces rays of light backwards from the eye into the scene, to see what objects they came from and therefore what color the light was.

Suppose one of these virtual rays of light is directed along the line $\mathbf{r}(t) = (0, -2, 0) + t\langle 0, 2, 2\rangle$. The plane $x - y + 2z - 10 = 0$ represents the side of a hill in the virtual world. Where does the ray intersect the hill?

Bonus. For up to 2 points of extra credit, suppose our computer animated snake is also part of the virtual world. It is sitting as it was back in Problem Set 1 — in particular, its head is the sphere $x^2 + y^2 + (z - 2)^2 = 1$. Where, if at all, does the ray intersect the snake's head? You may use Mathematica or calculators to help solve this problem; I recommend looking into Mathematica's "Solve" function.

Question 4.

Part A. (Exercise 34 in section 11.5E of our textbook.)

Show that there is no plane perpendicular to $\mathbf{n} = \langle 1, 1, 0\rangle$ that also contains points $(1, 2, 3)$ and $(2, 3, 4)$.

Part B. Describe a way you could determine whether an arbitrary line, $\mathbf{r}(t) = P + t\mathbf{v}$ is parallel to an arbitrary plane, $ax + by + cz + d = 0$. What could your answer have to do with solving Part A?

Question 5. Consider the plane $x + 2y - z = 4$ and the line $(1, 0, 0) + t\langle 1, 1, 0\rangle$. This line passes through the plane. Find the angle between the line and the plane when it does so. You may not use a calculator in this problem.

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.