Math 223 01
Prof. Doug Baldwin

Problem Set 4 — The Cross Product

Complete by **Wednesday, February 12**
Grade by **Friday, February 14**

Purpose

This problem set concentrates on building your understanding of the vector cross product. However, it also reinforces knowledge of the dot product, and begins developing an understanding of lines in three dimensions. By the time you finish this problem set I expect you to be able to...

- Calculate cross products
- Use dot products to solve problems involving perpendicularity
- Reason about perpendicular and parallel vectors, using dot products and/or cross products as appropriate
- Find equations for lines and determine when points are on those lines.

Background

Cross products are discussed in section 11.4 of our textbook, and were discussed in class on February 6. Dot products are in section 11.3 of the texbook, and in class notes from February 3 and 5. The first half of section 11.5 deals with lines, which we discussed in class on February 7 and 10.

Activity

Solve the following problems:

Question 1. (Based on problems 2 and 4 in section 11.4E of our textbook.)

For each of the following pairs of vectors, find the cross product $\mathbf{u} \times \mathbf{v}$. Express the cross products in either component form or in terms of standard unit vectors, whichever you prefer. Then sketch the vectors and the cross product (hand-drawn sketches are fine).

Part A. $\mathbf{u} = \langle 3, 2, -1 \rangle; \mathbf{v} = \langle 1, 1, 0 \rangle$.

Part B. $\mathbf{u} = 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}}; \mathbf{v} = 3\hat{\mathbf{i}} + \hat{\mathbf{k}}$

Question 2. One sometimes has a vector in 3 dimensions, and wants to find a more or less arbitrary vector perpendicular to it. (For example, if you want to build a coordinate system around some known principal direction, you could use a vector pointing in that direction as one of the axes, and an arbitrary vector perpendicular to it as a second axis. Bonus question: how would you then find the third axis?)

Phineas Phoole suggests that if you need a vector perpendicular to $\mathbf{v} = \langle x, y, z \rangle$, you can always use $\mathbf{u} = \langle -z, 0, x \rangle$. Is Phineas correct? Explain why or why not.

Question 3. Imagine a plane that contains points $P(1, 3, -1)$ and $Q(0, -7, 2)$. Furthermore suppose that vector $\mathbf{v} = \langle -3, 0, -1 \rangle$ is perpendicular to the plane. Find a third point, not colinear with (i.e., not in the same line as) P and Q, that is also in the plane. (Note: there are many ways to solve this problem.)

Question 4. Spacey the Space Traveler is playing zero-gravity space baseball. Spacey hits the ball when it is at point $(1, 1, -2)$, and sends it moving in direction $\langle -1, 10, 4 \rangle$. (If you care, I'm thinking of these things relative to Spacey's right-forward-up coordinate system, with distances measured in feet.) Because there is no gravity or air friction in zero-gravity space baseball, the ball will keep moving in this direction forever, unless one of the other players catches it.

Part A. Give an equation for the line along which the ball moves.

Part B. The center of the centerfielder's glove is at point $(-49, 501, 200)$, and doesn't move. Does the centerfielder catch Spacey's ball?

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.