Math 223 01
Prof. Doug Baldwin

Problem Set 3 — Vectors

Complete by **Sunday, February 9**
Grade by **Tuesday, February 11**

Purpose

This problem set reinforces your understanding of vectors. Specifically, by the time you finish this problem set I expect you to be able to…

- Use vectors to describe real-world values
- Perform basic arithmetic operations on vectors
- Describe and convert between vector representations
- Calculate dot products
- Use dot products to calculate angles and projections.

Background

This problem set exercises ideas from the second half of section 11.2, and all of 11.3, in our textbook. We discussed this material in classes between January 31 and February 5. Question 3 uses terminology related to video game viewing projections, which we defined in class on February 5.

Activity

Solve the following problems:

Question 1. It is a little-known, and even less-believed, "fact" that the Bronze Bear statue in Geneseo marks the origin of the universe's coordinate system. The universe's positive x axis runs south down the center of Main Street, the positive y axis runs east along Center Street, and the positive z axis runs up through the center of the statue.

Part A. Suppose you are pedaling a bicycle north along Main Street in such a manner that you create a force of 2 pounds pushing the bicycle forward. Describe this force as a vector (1) in component form, and (2) in terms of standard unit vectors $\hat{\mathbf{i}}$, $\hat{\mathbf{j}}$, and $\hat{\mathbf{k}}$.

Part B. Now suppose a wind kicks up, exerting force $\langle 1, 1, 0.2 \rangle$ (in pounds) on you. Describe the direction of this force in English. What is the net force acting on you (assume you keep pedaling as in Part A).

Part C. If the wind gets stronger so that it exerts twice the force it did in Part B, while still blowing in the same direction, what is the new net force on you?

Part D. About 4 years ago, a truck collided with the bear fountain, and severely damaged it (this part really happened). While the bear was being repaired, the Universe Distances and Coordinates Authority temporarily relocated the origin of the universe to an unused salt mine west of Geneseo (this part may not have happened exactly that way). The temporary origin was 3 miles west of the bear fountain, half a mile south, and half a mile below it. The temporary axes pointed in the same directions as the original ones did. Describe the relocation as a displacement vector, i.e., a vector whose components are the change in x, y, and z coordinates.

Part E. Let (x', y', z') denote the coordinates in the temporarily universe coordinate system of the point whose coordinates in the original system were (x, y, z). Give an equation involving the displacement vector from Part D that lets you calculate (x', y', z') from (x, y, z).

Question 2. (Inspired by a problem in an older version of our textbook.)

Water molecules consist of an oxygen atom and two hydrogen atoms. Suppose the oxygen atom in a certain water molecule is centered at the origin, and the hydrogen atoms are centered at points $(-1, 1, -1)$ and $(-1.06, -1.06, 0.87)$. What is the angle between the two oxygen-hydrogen bonds in this molecule? (Assume that the bonds lie along the lines connecting the centers of each hydrogen atom to the center of the oxygen atom.)

You may use a calculator to do the calculations for this problem, but set up those calculations by hand.

Question 3. The player in a first-person video game has a "right" vector $\mathbf{r} = \langle 1, 2, 1 \rangle$, expressed relative to the global coordinate system. A character in the video game is moving with velocity $\langle 3, 0, 0 \rangle$ meters per second relative to the global coordinate system. How fast is this character moving to the right, as seen by the player?

Question 4. Our textbook asserts, but doesn't prove, that dot product distributes over vector addition. Specifically, the book claims that if $\mathbf{u}$, $\mathbf{v}$, and $\mathbf{w}$ are 3-dimensional vectors, then $\mathbf{u} \cdot (\mathbf{v} + \mathbf{w}) = \mathbf{u} \cdot \mathbf{v} + \mathbf{u} \cdot \mathbf{w}$. Prove this claim. (Note: one way to go beyond what I expect on this problem would be to prove the claim for vectors of any size, not just 3 components.)

Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.