Math 223 01
Prof. Doug Baldwin

# Problem Set 2 — 3-Dimensional Coordinate Systems and Surfaces

Complete by **Monday, February 3**
Grade by **Wednesday, February 5**

## Purpose

This problem set reinforces your understanding of surfaces in 3 dimensions and the variety of coordinate systems that can be used to describe 3-dimensional space. Specifically, by the time you finish this problem set I expect you to be able to...

- Recognize when shapes are cylinders and quadric surfaces
- Recognize various shapes or regions in rectangular and cylindrical coordinate systems
- Convert both ways between rectangular and cylindrical coordinates

## Background

Cylinders and quadric surfaces are covered in section 11.6 of our textbook, and we talked about them in class on January 29. Cylindrical coordinates are the first part of section 11.7, and we discussed them in class on January 30. Rectangular coordinates for 3-dimensional space are covered in the first half of section 11.2, and were discussed in class on January 24.

## Activity

Solve the following problems:

**Question 1.** Phineas Phoole claims that a cone is a kind of cylinder, in the general mathematical sense of the word, because a cone can be described as a set of straight lines that pass through a curve, specifically a circle. Is Phineas right? Explain why or why not in a sentence or two.

**Question 2.** Imagine a cylinder in the colloquial sense of the word (i.e., a tube with a circular cross section) whose radius is 2 units and whose axis lies along the $y$ axis in some rectangular coordinate system. Is this surface a quadric surface? Justify your answer in terms of the definition of the quadric surfaces as surfaces defined by equations of the form $Ax^2 + By^2 + Cz^2 + Dxy + Exz + Fyz + Gx + Hy + Jz + K = 0$ for constants $A$, $B$, $C$, $D$, $E$, $F$, $G$, $H$, $J$, and $K$.

**Question 3.** Give an English description of the geometric shape or region made up of points whose coordinates are of the form $(2, 4, 1 + x)$, where $x$ ranges over all the real numbers.

**Question 4.** Consider the equation $z = x^2 + y^2$, which defines a paraboloid in rectangular coordinates.

**Part A.** What is the equation of that same paraboloid in cylindrical coordinates?

**Part B.** The point $(1, 2, 5)$ in rectangular coordinates lies on the paraboloid. What are the corresponding cylindrical coordinates for this point?

**Part C.** The point $(1, \frac{\pi}{3}, 1)$ in cylindrical coordinates also lies on the paraboloid. What are the corresponding rectangular coordinates for this point?

## Follow-Up

I will grade this exercise in a face-to-face meeting with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please bring a written solution to the exercise to your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting 15 minutes long, and schedule it to finish before the end of the "Grade By" date above. If you worked in a group on this assignment and the group has one collective solution, the whole group can schedule a single meeting with me.