Math 223 01
Prof. Doug Baldwin

Problem Set 12 — Multivariable Integration

Complete by **Wednesday, April 22**
Grade by **Monday, April 27**

Purpose

This problem set reinforces your understanding of definite integrals of multivariable functions. By the time you finish this problem set, I expect that you will be able to

- Integrate multivariable functions over rectangular regions
- Integrate multivariable functions over non-rectangular regions
- Interpret multivariable integrals as areas, volumes, etc.
- Use Mathematica to evaluate multivariable integrals.

Background

This problem set is based on sections 14.2, 14.3, and 14.5 of our textbook. We covered that material in classes between April 6 and 13. Note that in classes we combined section 14.5 (on triple integrals) with the sections on double integrals.

We discussed, or will discuss, evaluating integrals in Mathematica on April 13 or 15.

Activity

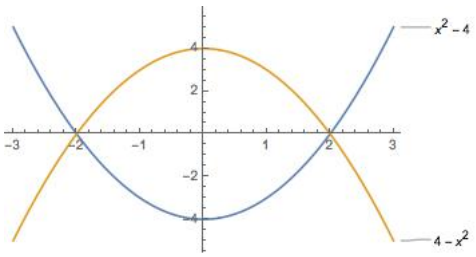
Solve the following problems.

Question 1. Evaluate

$$\int_0^1 \int_1^2 \int_0^2 x^2 y^2 z^2 \, dz \, dy \, dx$$

Check your answer by evaluating the integral with Mathematica.

Question 2. Let region D be the region between the curves $y = x^2 - 4$ and $y = 4 - x^2$, illustrated here:



Part A. Evaluate

$$\iint_D (x^2 + y)\, dA$$

over region D. Check your answer by also evaluating the integral with Mathematica.

Part B. Use a double integral to find the area of region D. Check your answer by also evaluating the integral with Mathematica.

Part C. *Bonus!* We didn't talk about average values of multivariable functions in class, although they are in the book. You therefore don't have to do this part, but can go beyond my expectations by doing so.

Find the average value of $x^2 + y$ over region D.

Question 3. One of the properties of double integrals that our textbook states is that over a rectangular region $R = [a, b] \times [c, d]$

$$\iint_R g(x)h(y)\, dA = \left(\int_a^b g(x)\, dx \right) \left(\int_c^d h(y)\, dy \right)$$

Justify this claim by showing how to express the double integral from the left side of the equation as an iterated integral and then rearrange it into the product on the right side.

Question 4. Evaluate

$$\int_0^3 \int_0^2 \int_0^1 \int_0^z dw\, dz\, dy\, dx$$

Give an interpretation of this integral as a "volume" (or, technically, hypervolume) that helps you make sense of the value you calculated.

Follow-Up

I will grade this exercise in a video chat, or alternative meeting, with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please have a written solution to the exercise available at your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting half an hour long, and schedule it to finish before the end of the "Grade By" date above. You must meet individually with me, even if you worked in a group on this problem set.

To "attend" your meeting, simply go to our course room in Canvas at the time you signed up for, and I will meet you there. I will set the room up so that you can share files from your computer with me.