Math 223 01

Prof. Doug Baldwin

Problem Set 11 — Applications of Partial Derivatives

Complete by **Sunday, April 12**

Grade by **Wednesday, April 15**

Purpose

This problem set reinforces your understanding of certain applications of partial derivatives, as well as of their generalizations as directional derivatives and gradients. By the time you finish this problem set, I expect that you will be able to

- Calculate, and solve problems using, directional derivatives
- Calculate and reason about gradients
- Find extreme values of multivariable functions
- Solve multivariable optimization problems by using Lagrange multipliers.

Background

This problem set is based on sections 13.6 through 13.8 of our textbook. We covered that material in classes between March 25 and April 2.

Activity

Solve the following problems.

Question 1. (Inspired by exercise 47 in the end-of-chapter exercises for section 13.6 in our book.)

The temperature at point (x, y, z) in a metal sphere is inversely proportional to distance from the origin.

Part A. If the temperature at point $(1, 2, 2)$ is 120° C, find the exact formula for temperature as a function of x, y, and z.

Part B. How fast is the temperature changing as one moves from point $(1, 2, 2)$ in the direction towards point $(2, 1, 3)$? Assume distances are in centimeters if you want to attach units to your answer.

Question 2. In many situations when one wants a single thing that acts as "the" derivative of a multivariable function, that single thing is the gradient. As such, you would expect gradients to behave in ways similar to derivatives. As an example of gradients behaving like derivatives, show that they obey a sum law analogous to the one for derivatives. More specifically, show that if $f(x, y)$ and $g(x, y)$ are multivariable functions, then

$$\nabla(f + g)(x, y) = \nabla f(x, y) + \nabla g(x, y) \tag{1}$$

As a way of doing slightly more than I expect, make your argument general enough to apply to functions of any number of variables.

Question 3. (Exercise 9 in the end-of-chapter exercises for section 13.7 of our book.)

Find the critical points of $f(x, y) = -x^3 + 4xy - 2y^2 + 1$ and use the second derivative test to classify them as minima, maxima, or saddle points.

Question 4. (From a problem suggested in the end-of-chapter exercises for section 13.8 of our textbook.)

A rectangular box with no top (i.e., a box with 4 sides and a bottom) is made from 12 square feet of cardboard. Use Lagrange multipliers to find the maximum volume the box can have. What are the dimensions of this box?

Follow-Up

I will grade this exercise in a video chat, or alternative meeting, with you. During this meeting I will look at your solution, ask you any questions I have about it, answer questions you have, etc. Please have a written solution to the exercise available at your meeting, as that will speed the process along.

Sign up for a meeting via Google calendar. Please make the meeting half an hour long, and schedule it to finish before the end of the “Grade By” date above. You must meet individually with me, even if you worked in a group on this problem set.

To “attend” your meeting, simply go to our course room in Canvas at the time you signed up for, and I will meet you there. I will set the room up so that you can share files from your computer with me.